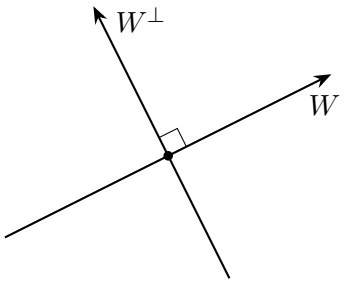
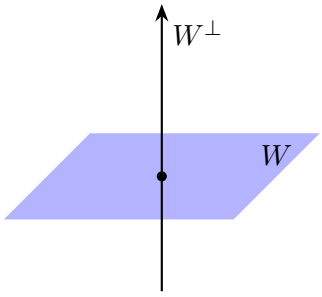
Definitions

- We say that a vector $\mathbf{x}$ is **orthogonal to a subspace** W of $\mathbb{R}^n$ if $\mathbf{x} \cdot \mathbf{w} = 0$ for all $\mathbf{w}$ in W.
- We say that two subspaces V and W are orthogonal if $\mathbf{v} \cdot \mathbf{w} = 0$ for every $\mathbf{v} \in V$ and every $\mathbf{w} \in W$.
- For a subspace W of $\mathbb{R}^n$, the **orthogonal complement** of W, denoted $W^\perp$, is the set of all vectors in $\mathbb{R}^n$ that are orthogonal to W. In set notation,

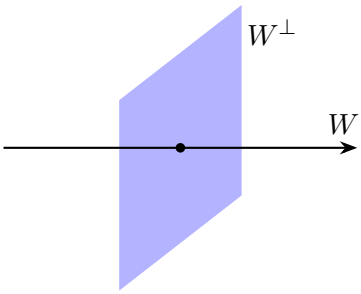
$$W^\perp = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{x} \cdot \mathbf{w} = 0 \text{ for all } \mathbf{w} \in W\}.$$



In $\mathbb{R}^2$



In $\mathbb{R}^3$



In $\mathbb{R}^3$