

LOG 10
SPG 15
①

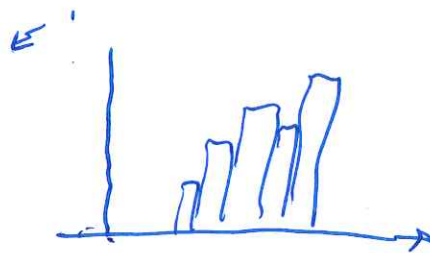
I) DERIVATION $\hat{=}$ CANONICAL
DISTRIBUTION (FN) (Sec 3.1 text)

A) $P(E_r) = P(E')$ $\Leftarrow$ prob. of observing
system in
a large reservoir
w/ energy state
 E_r $\equiv P(E_r) =$
prob. of observing
res in state $E' \equiv$
 $P(E')$

a) THIS FOLLOWS SINCE

$$E^0 = E' + E_r \quad \&$$

$\Rightarrow$ any time we obs. a
given E' , we instantly
know E_r



No.
Observations
in res.



No. of
obs
of
 E_r

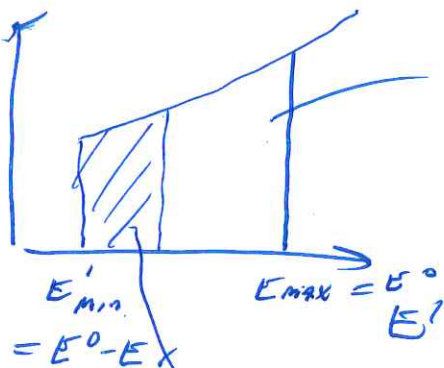
(implied
since
 $E_r = E^0 - E'$)

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B) $P(E') \propto \Omega'(E')$

prob. of E' given Θ $E' \gg E_r$

a) $\frac{\partial \Omega'}{\partial E'}$



$$A_{\text{TOT}} = \int_{E_{\min}}^{E_{\max}} \frac{\partial \Omega'}{\partial E'} dE'$$

$$= \Omega'(E_{\max}) - \Omega'(E_{\min})$$

$$A(E') = \int_{E_{\min}}^{E'} \frac{\partial \Omega'}{\partial E'} dE'$$

$$= \Omega'(E') - \Omega'(E_{\min})$$

b) $\therefore$ prob of
res. being ~~having~~ observing res.
in an energy state
between $E_{\min}$ and E'

$$is \equiv P(E') = \frac{A(E')}{A_{\text{TOT}}}$$

$$= \frac{\Omega'(E') - \Omega'(E_{\min})}{\Omega'(E_{\max}) - \Omega'(E_{\min})}$$

$$= c_1 \Omega'(E') + c_2$$

b1) $\therefore$

$$\boxed{P(E') \propto \Omega'(E')} \quad \checkmark$$

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c) Form $\ln \Omega'(\epsilon')$ $\equiv B'$ ch. 1

$$\ln(\Omega'(\epsilon')) = \ln \Omega(\epsilon_0) + \left. \frac{\partial \ln \Omega'}{\partial \epsilon'} \right|_{\epsilon_0} (\epsilon' - \epsilon_0)$$

$$\therefore \Omega'(\epsilon') = A_0 \exp(-B'\epsilon')$$

d) BUT AT equil., we know
(ch. 1) that

$$B = B'$$

$$\left(\frac{\partial \ln \Omega(\epsilon_r)}{\partial \epsilon} \right)_{N, V} = \left(\frac{\partial \ln \Omega'(\epsilon')}{\partial \epsilon'} \right)_{N', V'}$$

$$\therefore \text{b1) } \oplus \text{ c) } \Rightarrow$$

$$P(\epsilon_r) = \Omega'(\epsilon') \\ = A_0 \exp(-B\epsilon_r)$$

e) Normalise $P(\epsilon_r)$ to get A_0 :

$$\sum_i P(\epsilon_i) = 1 = A_0 \sum_i e^{-B\epsilon_i}$$

$$\therefore A_0 = 1/Q$$

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$$\therefore \boxed{P(\epsilon_i) = \frac{1}{Q} e^{-\beta \epsilon_i}}$$

$$Q \equiv \sum_i e^{-\beta \epsilon_i}$$

II) Derivation 2 of Canonical distribution

A) First move to continuous parameters

$$W(\epsilon_m) = \frac{\tilde{N}!}{n_1! n_2! \dots n_m!}$$

where
 m
 gotten
 from

$M E_{min} = \tilde{E} = \text{fixed total energy of ensemble}$

$E_{min} = \text{min / lower quantum energy state}$

$W(\) = \text{no of ways that a given set of system energy states:}$

$(n_i = \text{no of systems in given state } i, \text{ etc.})$

$U = 10$

$SPQ = 15$

can arise in an ensemble of
at $\tilde{N}$ ^{replicas} copies of our system

1a) Follow rough notes (from
Soutag Van Wylen) to
show that

$$W = \frac{\tilde{N}!}{n_1! n_2! \dots}$$

BA

B) Now, I'll use a partial
(very partial) inductive
proof of Pathria's
claim that $W(E, N, S)$ is
max'd ~~when~~ @ equilibrium

i) Let $\boxed{\tilde{N} = 5, \tilde{E} = 6} \rightarrow$ both fixed
At any ~~of~~ instant, when
we observe our 5 member

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ensemble, we'll see the
 5 systems in various (pure)
 Quantum energy states.

Observation 1

Let B denote a sys. in
 state w/ energy = 2;

Let A =
 state w/
 energy = 1

	B	A	A	A	A
Er	2	1	1	1	1

⇒ NOTE $\tilde{E} = 2 + 1 + 1 + 1 + 1$
 $= 6 \checkmark$

Here $W() = \frac{5!}{1! 4!} = \frac{120}{24} = \boxed{5}$

$\uparrow \quad \uparrow$
 $n_2! \quad n_1!$

Obs 2:

	B	B	B	0	0
Er	2	2	2	0	0

⇒ $\tilde{E} = 6$
 $n_2 = 3$
 $n_1 = 0$

⇒ $W() = \frac{5!}{3! 0!} = \frac{120}{6} = \boxed{20}$

Let 10
30. 15
(9)

obs 3

R B A A

or 2 2 1 1

$$\Rightarrow \tilde{G} = 6 \rightarrow w = \frac{5!}{2!2!} = \frac{\tilde{n}!}{n_1!n_2!}$$

$$= \frac{120}{4} = \boxed{30}$$

~~7~~ I don't think there are any more possibilities (for $\tilde{G} = 6$).

1) So, the total number of possible observations of our 5-member ensemble
 $= 5 + 20 + 30 = \boxed{55}$

2) $\therefore$ prob. of observing ensemble in one of the 5 distns ass'd w/ obs. 1

$$= \frac{5}{55} = p_1$$

2a) prob. for observing
one of the 20
distr ~~set~~
_{subset}
ass'd w/ obs. 2

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$$= \frac{20}{55}$$

1b) similar
obs 3 = $\frac{30}{55}$

3) So, FINALLY, ASSUMING

THAT THE MOST PROBABLE
DISTRIBUTION SET - IN OUR

CASE THE ONE ASS'D W/

OBSERV. 3 - ~~OR~~ IS THE ONE

THAT APPEARS WHEN

equilibrium sets in, THEN

WE HAVE PROVEN (at

least made a good

start at proving) that

W IS MAXIMIZED when equilibrium
sets in