1. Suppose that $A = \begin{bmatrix} 2 & 0 & -2 & -4 \\ -2 & -1 & 1 & 2 \\ 0 & -1 & -1 & -2 \end{bmatrix}$.

   (a) Is the vector $\begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}$ in Col(A)?

   (b) Is the vector $\begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}$ in Col(A)?

   (c) Is the vector $\begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$ in Nul(A)?

   (d) Is the vector $\begin{bmatrix} 1 \\ -1 \\ 3 \\ -1 \end{bmatrix}$ in Nul(A)?

   (e) Is the vector $\begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix}$ in Nul(A)?

2. Find an example of each of the following.

   (a) A 3×3 matrix A and a nonzero vector $\mathbf{b}$ such that $\mathbf{b}$ is in Col(A) but $\mathbf{b}$ is not equal to any of the columns of A.

   (b) A 3×3 matrix A and a nonzero vector $\mathbf{b}$ such that $\mathbf{b}$ is NOT in Col(A).

   (c) A 3×3 matrix A and a nonzero vector $\mathbf{b}$ such that $\mathbf{b}$ is in Nul(A).

3. Suppose F is a 5×5 matrix whose column space is not equal to $\mathbb{R}^5$. What can you say about Nul(F)?

4. What can you say about Nul(C) if C is a 5×4 matrix with linearly independent columns?

5. What are the possible values of rank(B) if B is a 6×10 matrix. What are the possible values of nullity(B)?

6. Find rank(M) and nullity(M) where is the multiplication table matrix, shown below.

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 \\ 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24 & 27 \\ 4 & 8 & 12 & 16 & 20 & 24 & 28 & 32 & 36 \\ 5 & 10 & 15 & 20 & 25 & 30 & 35 & 40 & 45 \\ 6 & 12 & 18 & 24 & 30 & 36 & 42 & 48 & 54 \\ 7 & 14 & 21 & 28 & 35 & 42 & 49 & 56 & 63 \\ 8 & 16 & 24 & 32 & 40 & 48 & 56 & 64 & 72 \\ 9 & 18 & 27 & 36 & 45 & 54 & 63 & 72 & 81 \end{bmatrix}$$

7. Find rank(C) and nullity(C) where

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

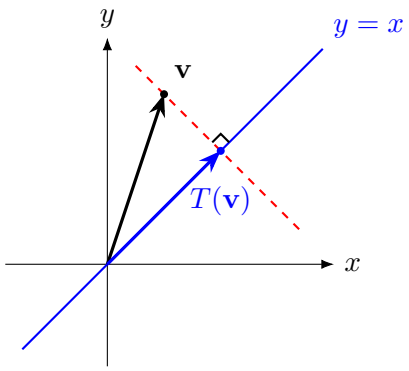
8. Find a basis for the range and kernel of each linear transformation.

   (a) $R : \mathbb{R}^2 \to \mathbb{R}^2$ is the rotation counterclockwise by $\dfrac{\pi}{3}$ radians.

   (b) $S : \mathbb{R}^2 \to \mathbb{R}^2$ is the reflection over the x-axis.

   (c) $T : \mathbb{R}^3 \to \mathbb{R}$ given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = x + z$

(d) $P : \mathbb{R}^2 \to \mathbb{R}^2$ is the projection onto $y = x$



9. Explain why the following inequalities are true.

(a) $\text{rank}(AB) \leq \min(\text{rank}(A), \text{rank}(B))$

(b) $\text{nullity}(AB) \geq \text{nullity}(B)$