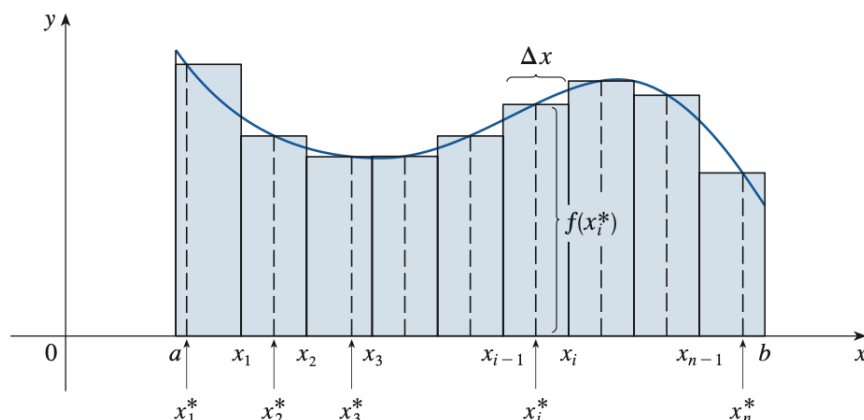


### Single Integrals

Recall:  $\int_a^b f(x) dx$  computes the signed area under the graph of  $f(x)$ . We define this using Riemann sums.



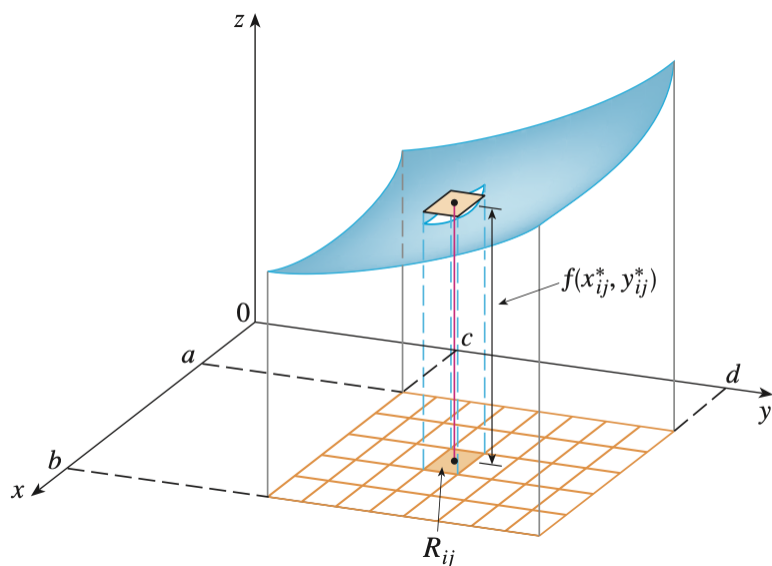
$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x$$

Here  $\Delta x$  is the length of the subintervals. By taking equal length subintervals, we have that  $\Delta x = \frac{b-a}{n}$ . Also,  $x_i^*$  is a sample point in the  $i$ th subinterval,  $[x_{i-1}, x_i]$ .

### Double Integrals

For  $f(x, y)$ , we'll define the double integral over a region  $R$  in the  $xy$ -plane using double Riemann sums. The double integral of  $f(x, y)$  is denoted  $\iint_R f(x, y) dA$ .

We first focus on rectangular regions  $R = [a, b] \times [c, d]$ , which we divide into subrectangles  $R_{ij}$ .



$$\iint_R f(x, y) dA \text{ computes the signed volume under the graph of } f(x, y)$$

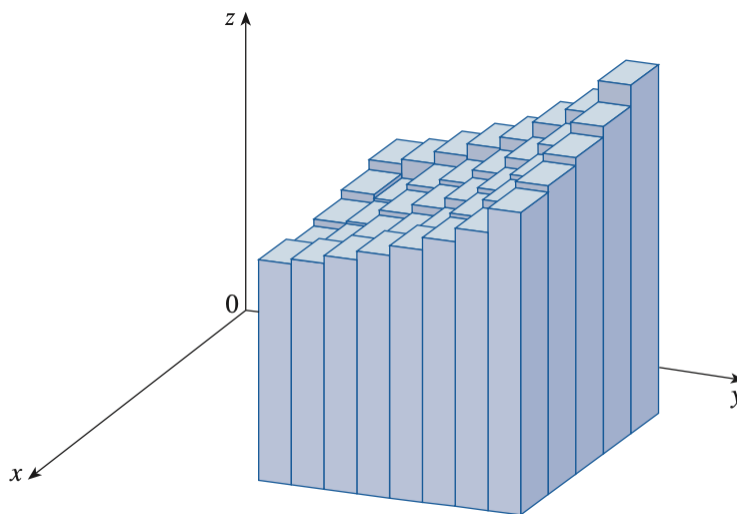
Each subrectangle  $R_{ij}$  has area  $\Delta A = \Delta x \Delta y = \left(\frac{b-a}{m}\right) \left(\frac{c-d}{n}\right)$ .

The point  $(x_{ij}^*, y_{ij}^*)$  is the sample point for  $R_{ij}$ . The height of the  $ij$ th approximating box is  $f(x_{ij}^*, y_{ij}^*)$ .

The volume of the  $ij$ th approximating box is  $V_{ij} = f(x_{ij}^*, y_{ij}^*) \Delta A$ .

We sum  $V_{ij}$  for each  $1 \leq i \leq m$  and each  $1 \leq j \leq n$  to approximate the volume under the graph of  $f(x, y)$ . The result is called a double Riemann sum.

$$\text{Volume under } f(x, y) \approx \sum_{i=1}^m \sum_{j=1}^n V_{ij} = \underbrace{\sum_{i=1}^m \sum_{j=1}^n f(x_{ij}^*, y_{ij}^*) \Delta A}_{\text{Double Riemann sum}}$$



We get the the double integral of  $f(x, y)$  over the rectangular region  $R$  by taking the limit as both  $m$  and  $n$  approach infinity:

$$\iint_R f(x, y) dA = \lim_{m, n \rightarrow \infty} \sum_{i=1}^m \sum_{j=1}^n f(x_{ij}^*, y_{ij}^*) \Delta A.$$