

1A

#1 $u=x$ $dv=\sec^2 x$
 $du=dx$ $v=\tan x$

$$\int x \sec^2 x dx = x \tan x - \int \tan x dx$$

* solve by u-sub

#2 $\int \frac{e^x + 1}{e^x + x} dx$ $u=e^x + x$ $\int \frac{1}{u} du$
 $du=(e^x + 1) dx$

#3 $\int \sin^{10} x \cos^5 x dx =$ odd cosine

$$\int \sin^{10} x \cos^4 x \cos x dx =$$

$$\int \sin^{10} x (\cos^2 x)^2 \cos x dx = \int \sin^{10} x (1 - \sin^2 x)^2 \cos x dx$$

$$u = \sin x \quad du = \cos x dx \quad \int u^{10} (1 - u^2)^2 du$$

#4 $\int \tan^6 x \sec^5 x dx$ odd tangent

$$\int \tan^6 x \sec^4 x \sec x \tan x dx =$$

$$\int (\tan^2 x)^3 \sec^4 x \sec x \tan x dx =$$

$$\int (\sec^2 x - 1)^3 \sec^4 x \sec x \tan x dx \quad \begin{array}{l} u = \sec x \\ du = \sec x \tan x dx \end{array}$$

$$\boxed{\int (u^2 - 1)^3 u^4 du}$$

#5 $\int x^2 \sqrt[3]{x^3 + 5} dx = \int x^2 (x^3 + 5)^{1/3} dx$

u-sub $u = x^3 + 5 \quad du = 3x^2 dx \quad \frac{1}{3} du = x^2 dx$

$$\frac{1}{3} \int u^{1/3} du = \frac{1}{3} \frac{u^{4/3}}{4/3} = \frac{1}{3} \cdot \frac{3}{4} u^{4/3} =$$

$$\frac{1}{4} u^{4/3} \rightarrow \boxed{\frac{1}{4} (x^3 + 5)^{4/3} + C}$$

$$\underline{\#6} \quad \int \frac{\ln x}{\sqrt[3]{x}} dx = \int \frac{\ln x}{x^{1/3}} dx = \int x^{-1/3} \ln x dx$$

by parts $u = \ln x \quad dv = x^{-1/3} dx$

$$du = \frac{1}{x} dx \quad v = \frac{3}{2} x^{2/3}$$

$$\int x^{-1/3} \ln x dx = \frac{3}{2} x^{2/3} \ln x - \int \frac{3}{2} x^{2/3} \cdot \frac{1}{x} dx =$$

$$\frac{3}{2} x^{2/3} \ln x - \frac{3}{2} \int x^{-1/3} dx =$$

$$\frac{3}{2} x^{2/3} \ln x - \frac{3}{2} \frac{x^{2/3}}{2/3} = \frac{3}{2} x^{2/3} \ln x - \frac{9}{4} x^{2/3} + C$$

by parts

$$\underline{\#7} \quad \int \cos(\ln x) dx \quad u = \cos(\ln x) \quad dv = 1 dx$$

$$du = -\sin(\ln x) \frac{1}{x} dx \quad v = x$$

$$\int \underbrace{\cos(\ln x)}_{u} \underbrace{dx}_{dv} = \underbrace{x \cos(\ln x)}_{uv} - \int \underbrace{-\sin(\ln x)}_{v} \underbrace{\frac{1}{x} \cdot x}_{du} dx$$

$$\int \cos(\ln x) dx = x \cos(\ln x) + \int \sin(\ln x) dx \quad *$$

by parts

$$\int \sin(\ln x) dx \quad \begin{array}{ll} u = \sin(\ln x) & dv = 1 dx \\ du = \cos(\ln x) \frac{1}{x} dx & v = x \end{array}$$

$$\int \underbrace{\sin(\ln x)}_{u} \underbrace{dx}_{dv} = \underbrace{x \sin(\ln x)}_{u v} - \int \underbrace{\cos(\ln x) \frac{1}{x}}_{v du} \cdot x dx$$

$$\int \sin(\ln x) dx = x \sin(\ln x) - \int \cos(\ln x) dx$$

$$* \int \cos(\ln x) dx = x \cos(\ln x) + x \sin(\ln x) - \int \cos(\ln x) dx$$

Add $\int \cos(\ln x) dx$ to both sides

$$\int \cos(\ln x) dx + \int \cos(\ln x) dx = x \cos(\ln x) + x \sin(\ln x) - \int \cos(\ln x) dx + \int \cos(\ln x) dx$$

$$2 \int \cos(\ln x) dx = x \cos(\ln x) + x \sin(\ln x)$$

$$\int \cos(\ln x) dx = \frac{1}{2} x \cos(\ln x) + \frac{1}{2} x \sin(\ln x) + C$$

#8 $\int_0^{\pi/2} \sin^2 x \cos^5 x dx$

odd cosine

$$\int_0^{\pi/2} \sin^2 x \cos^4 x \cos x dx = \int_0^{\pi/2} \sin^2 x (\cos^2 x)^2 \cos x dx =$$

$$\int_0^{\pi/2} \sin^2 x (1 - \sin^2 x)^2 \cos x dx$$

u-sub

$$u = \sin x$$

$$du = \cos x dx$$

$$x=0 \quad u=0$$

$$x=\pi/2 \quad u=\sin \pi/2 = 1$$

$$\int_0^1 u^2 (1-u^2)^2 du =$$

$$\int_0^1 u^2 (1 - 2u^2 + u^4) du = \int_0^1 (u^2 - 2u^4 + u^6) du =$$

$$\left. \frac{1}{3} u^3 - \frac{2}{5} u^5 + \frac{1}{7} u^7 \right|_0^1 = \frac{1}{3} - \frac{2}{5} + \frac{1}{7} =$$

$$\frac{35 - 42 + 15}{3 \cdot 5 \cdot 7} = \frac{8}{105}$$