

Turn in these problems:

1. Given $a_n = \frac{7n+3}{\sqrt{n^5-4}}$, consider the series $\sum_{n=2}^{\infty} a_n = \sum_{n=2}^{\infty} \frac{7n+3}{\sqrt{n^5-4}}$

Given the following arguments for the convergence or divergence of the series. Determine if **each** argument is valid or invalid. Explain your reasoning.

(a) $\lim_{n \rightarrow \infty} a_n = 0$. Therefore, $\sum_{n=2}^{\infty} a_n$ converges by the Test for Divergence.

WRONG!

big example: $\sum \frac{1}{n}$ diverges but $\lim_{n \rightarrow \infty} a_n = 0$

invalid

(b) Let $b_n = \frac{7n}{\sqrt{n^5}}$. Then $0 \leq a_n \leq b_n$. Therefore, $\sum_{n=2}^{\infty} a_n$ converges by the Comparison Test.

$\sum_{n=2}^{\infty} b_n = \sum_{n=2}^{\infty} \frac{7}{n^{5/2-1}} = \sum_{n=2}^{\infty} \frac{7}{n^{3/2}}$ $p = \frac{3}{2} > 1$

Is $a_n \leq b_n$ for all n ?

$\frac{7n+3}{\sqrt{n^5-4}} \leq \frac{7n}{\sqrt{n^5}}$ ← NOT true

invalid

(c) Let $b_n = \frac{7n}{\sqrt{n^5}}$. Then $0 \leq b_n \leq a_n$. Therefore, $\sum_{n=2}^{\infty} a_n$ diverges by the Comparison Test.

comparing to a convergent series, so can't get diverges.

valid

(d) Let $b_n = \frac{7n}{\sqrt{n^5}}$. Then $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c$, where c is a ^{positive} finite value. Therefore, by the Limit Comparison Test $\sum_{n=2}^{\infty} a_n$ converges since $\sum_{n=2}^{\infty} b_n$ converges.

$\lim_{n \rightarrow \infty} \frac{7n+3}{\sqrt{n^5-4}} \cdot \frac{\sqrt{n^5}}{7n} = \lim_{n \rightarrow \infty} \left(\frac{7n+3}{7n} \right) \left(\frac{\sqrt{n^5}}{\sqrt{n^5-4}} \right) = 1 \cdot 1$

invalid

(e) Let $b_n = \frac{7n}{\sqrt{n^5}}$. Then $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c$, where c is a finite value. Therefore, by the Limit Comparison Test $\sum_{n=2}^{\infty} a_n$ diverges since $\sum_{n=2}^{\infty} b_n$ diverges.

↑
false

2. Explain why the Comparison Test can or cannot be used to decide if the series converges or diverges. You do not have to determine if the series converges or diverges.

← positive terms, if the comparison works

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

(b) $\sum_{k=1}^{\infty} \frac{\ln(k)}{k}$

(c) $\sum_{n=1}^{\infty} \sin(n)$

← Remember to check for positive terms and that $b_n \geq 0$.

3. Use the Comparison Test or Limit Comparison Test to determine whether each series is convergent or divergent. Carefully show your work and justify your conclusion.

(a) $\sum_{k=1}^{\infty} \frac{4k^2 - k}{k^3 + k + 1}$

(b) $\sum_{k=0}^{\infty} \frac{1 + \cos k}{8^k}$

(c) $\sum_{k=1}^{\infty} \frac{1}{k^2 \arctan(k)}$

Pick $b_k = \frac{4k^2}{k^3} \geq 0$. $b_k = \frac{2}{8^k}$

or
 $b_k = \frac{1}{8^k}$

$b_k = \frac{1}{k^2 (\frac{\pi}{2})}$

or
 $b_k = \frac{1}{k^2}$

4. Let c_k be the digits of π . That is, $c_0 = 3, c_1 = 1, c_2 = 4$, etc. Show that the series converges.

$\sum_{k=0}^{\infty} \frac{c_k}{10^k} = 3.14...$

Similar problem in notes (Tues. Week 8)

5. Use the expressions below to determine:

(a) If the sequence $\{a_n\}_{n=1}^{\infty}$ converges or diverges.

← take the limit

(b) If the series $\sum_{n=1}^{\infty} a_n$ converges or diverges.

← apply a test

I. $a_n = \frac{e^n}{1 + e^{2n}}$

III. $a_n = \frac{n}{1 + n^2}$

II. $a_n = \frac{\sqrt{n}}{1 + \sqrt{n}}$

V. $a_n = 2 + (-1)^n$

6. Explain why the Ratio Test can or cannot be used to determine whether the series is convergent.

(a) $\sum_{n=1}^{\infty} (-1)^n$

(c) $\sum_{n=1}^{\infty} \frac{2^n + 1}{n2^n - 1}$

(b) $\sum_{n=1}^{\infty} \sin(n)$

(d) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

Issues: get $L = 1$

OR $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| \text{ DNE}$

7. Explain why the Alternating Series Test can or cannot be used to determine whether the series is convergent.

(a) $\frac{2}{1} - \frac{1}{1} + \frac{2}{2} - \frac{1}{2} + \frac{2}{3} - \frac{1}{3} + \dots$

(c) $\sum_{n=1}^{\infty} (-1)^{n-1} \left(2 - \frac{1}{n} \right)$

(b) $\sum_{n=2}^{\infty} (-1)^n \sin(n)$

(d) $\sum_{n=2}^{\infty} \frac{(-1)^n}{e^2}$

are the 2 conditions of AST satisfied?

8. Determine whether each series is absolutely convergent, conditionally convergent or divergent. Carefully show your work.

Wed. notes

(a) $\sum_{n=1}^{\infty} \frac{n!}{99^n}$ $\leftarrow$ Div. Test
 $\lim_{n \rightarrow \infty} a_n = \infty$

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 1}$

(c) $\sum_{k=1}^{\infty} \frac{\sin 4k}{2^k}$

(d) $\sum_{k=1}^{\infty} k \left(\frac{5}{6}\right)^k$

(e) $\sum_{k=1}^{\infty} \frac{(-3)^k k!}{(2k)!}$

(f) $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{\sqrt{n}}$

9. Test each of the following series for convergence or divergence. Carefully explain your reasoning.

(a) $\overset{a_1}{-\frac{3}{4}} + \overset{a_2}{\frac{5}{5}} - \overset{a_3}{\frac{7}{6}} + \overset{a_4}{\frac{9}{7}} - \overset{a_5}{\frac{11}{8}} + \dots$

(b) $\sum_{k=3}^{\infty} (-1)^k \frac{k}{\sqrt{k^3 + 1}}$

(c) $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{\ln(k+5)}$

(d) $\sum_{k=1}^{\infty} (-1)^k \cos\left(\frac{\pi}{k}\right)$

$\rightarrow a_n = (-1)^n \cdot \frac{(1+2n)}{(3+n)}$

$\lim_{n \rightarrow \infty} \underbrace{(-1)^n}_{\text{alternates}} \cdot \underbrace{\frac{1+2n}{3+n}}_{\downarrow 2}$ DNE