

Exam open 7-10pm, 1 hour

topics: u-sub

IBP

trig integrals

} do a bunch of integrals

Practice exams on Canvas

Written HW #11)

$$\int \sin(ax) \cos(bx) dx$$

$$= \frac{1}{b^2 - a^2} (b \sin(ax) \sin(bx) + a \cos(ax) \cos(bx)) + C$$

(a) Product Identity:

$$\sin A \cos B = \frac{1}{2} (\sin(A+B) + \sin(A-B))$$

* (c) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

try to get your answer in the form

$$\frac{1}{2(b^2 - a^2)} ((a-b) \cos(ax+bx) + (a+b) \cos(ax-bx)) + C$$

Trig Integrals - tan/sec

$$\int \tan^n x \sec^m x \, dx$$

Goal: do a u-sub with

$$u = \sec x$$

$$du = \sec x \tan x \, dx$$

↑
for this to work,
need a factor of
 $\sec x \tan x$ and
everything else
in terms of $\sec x$

OR

$$u = \tan x$$

$$du = \sec^2 x \, dx$$

↑
for this to
work, need a
factor of $\sec^2 x$
and everything
else in terms
of $\tan x$

Remember: we can use $\tan^2 x + 1 = \sec^2 x$
to convert even powers of $\tan x$
to $\sec x$ and vice versa.

ex: $\int \tan^5 x \sec^2 x \, dx$

~~$u = \sec x$
 $du = \sec x \tan x \, dx$~~ OR

$u = \tan x$
 $du = \sec^2 x \, dx$

$$\begin{aligned} &= \int u^5 \, du \\ &= \frac{1}{6} u^6 + C \\ &= \frac{1}{6} \tan^6 x + C \end{aligned}$$

(Note: $u = \sec x$ also works, but it's a lot more work)

ex: $\int \tan^3 x \sec x \, dx$

$u = \sec x$
 $du = \sec x \tan x \, dx$

OR

~~$u = \tan x$
 $du = \sec^2 x \, dx$~~

↗
won't work,
can only convert
even powers of
 $\tan x$ to $\sec x$,
can't get $\sec^2 x$

$$\int \tan^3 x \cdot \sec x dx = \int \tan^2 x \cdot \tan x \sec x dx$$

$$= \int (\sec^2 x - 1) \cdot \tan x \sec x dx$$

$$u = \sec x$$

$$du = \sec x \tan x dx$$

$$= \int u^2 - 1 du$$

$$= \frac{1}{3} u^3 - u + C$$

$$= \frac{1}{3} \sec^3 x - \sec x + C$$

ex: $\int \tan^2 x \sec^4 x dx$

~~$$u = \sec x$$~~
~~$$du = \sec x \tan x dx$$~~

↗
doesn't work

OR

$$u = \tan x$$

$$du = \sec^2 x dx$$

$$\int \tan x \sec^3 x \cdot \underbrace{\sec x \tan x dx}_{du}$$

↑
can't convert
to sec x

$$= \int \tan^2 x \cdot \underbrace{\sec^2 x}_{\text{convert}} \cdot \underbrace{\sec^2 x}_{du} dx$$

$$= \int \tan^2 x (\tan^2 x + 1) \cdot \sec^2 x dx$$

$$\boxed{\begin{array}{l} u = \tan x \\ du = \sec^2 x dx \end{array}} \leftarrow$$

$$= \int u^2 (u^2 + 1) du$$

$$= \int u^4 + u^2 du$$

$$= \frac{1}{5} u^5 + \frac{1}{3} u^3 + C$$

$$= \frac{1}{5} \tan^5 x + \frac{1}{3} \tan^3 x + C$$

ex: $\int_0^{\pi/4} \sec^6 x \, dx$

~~$u = \sec x$
 $du = \sec x \tan x \, dx$~~

OR $\boxed{\begin{array}{l} u = \tan x \\ du = \sec^2 x \, dx \end{array}}$

↗ can only get $\tan^2 x$ by converting $\sec x$, which is too many

$$= \int_0^{\pi/4} \sec^4 x \cdot \underbrace{\sec^2 x \, dx}_{du}$$

$$= \int_0^{\pi/4} (\sec^2 x)^2 \sec^2 x \, dx$$

$$= \int_0^{\pi/4} (\tan^2 x + 1)^2 \sec^2 x \, dx$$

$$\begin{array}{l} u = \tan x \\ du = \sec^2 x \, dx \end{array}$$

bounds
 $x = 0 \Rightarrow u = \tan(0) = 0$
 $x = \pi/4 \Rightarrow u = \tan(\pi/4) = 1$

$$= \int_0^1 (u^2 + 1)^2 \, du$$

$$\begin{aligned}
&= \int_0^1 u^4 + 2u^2 + 1 \, du \\
&= \left(\frac{1}{5} u^5 + \frac{2}{3} u^3 + u \right) \Big|_0^1 \\
&= \frac{1}{5} + \frac{2}{3} + 1 - 0 \\
&= \frac{3 + 2 \cdot 5 + 15}{15} = \frac{28}{15}
\end{aligned}$$

Another strategy: write $\tan x$ & $\sec x$ in terms of $\sin x$ & $\cos x$

$$\tan x = \frac{\sin x}{\cos x}, \quad \sec x = \frac{1}{\cos x}$$

$$\text{ex: } \int \tan x \sec^3 x \, dx = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos^2 x} \, dx$$

$$= \int \frac{\sin x}{\cos^3 x} \, dx$$

$$\left(\begin{array}{l} u = \cos x \\ du = -\sin x \, dx \end{array} \right)$$

$$= \int \frac{1}{u^3} (-1) \, du$$

$$\int \frac{1}{u} du = \ln|u| + C$$

$$\int \frac{1}{u^2} du = \int u^{-2} du$$
$$= -u^{-1} + C$$

$$= - \int u^{-4} du$$

$$= -(-\frac{1}{3})u^{-3} + C$$

$$= \frac{1}{3} \cdot \frac{1}{u^3} + C$$

$$= \frac{1}{3} \cdot \frac{1}{\cos^3 x} + C$$

$$= \frac{1}{3} \sec^3 x + C$$

ex: $\int \frac{\tan^3 x}{\sec^2 x} dx = \int \frac{\frac{\sin^3 x}{\cos^3 x}}{\frac{1}{\cos^2 x}} dx$

$$= \int \frac{\sin^3 x}{\cancel{\cos^3 x}} \cdot \frac{\cancel{\cos^2 x}}{1} dx$$

~~cos x~~

$$= \int \frac{\sin^3 x}{\cos x} dx$$

$$= \int \frac{\sin^2 x}{\cos x} \cdot \sin x dx$$

$$u = \cos x$$

$$du = -\sin x dx$$

$$= \int \frac{1 - \cos^2 x}{\cos x} \cdot \sin x dx$$

$$= \int \frac{1 - u^2}{u} \cdot (-1) du$$

$$= \int \frac{u^2 - 1}{u} du \quad \leftarrow$$

$$= \int \frac{u^2}{u} - \frac{1}{u} du$$

$$= \int u - \frac{1}{u} du$$

$$= \frac{1}{2} u^2 - \ln|u| + C$$

$$= \frac{1}{2} \cos^2 x - \ln|\cos x| + C$$