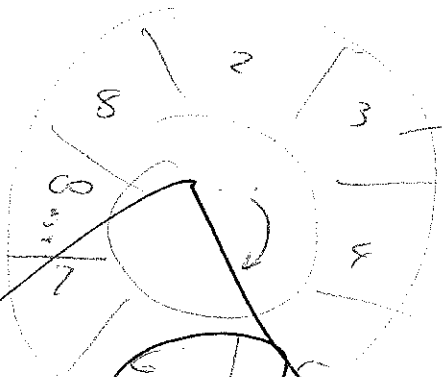


F) PROOF OF CONSTANCY OF K

LEC. 3
SP4 13

SYS 1
 N_1, V_1, E_1



Material 3
 N_3, V_3, E_3
etc

1) Apply the same arg as before

$$\Omega = \Omega_1(\omega_1) \Omega_m(\omega_m)$$

2) $\Rightarrow$ @ equilibrium

$$\left. \frac{\partial \ln \Omega_1(\omega_1)}{\partial E_1} \right|_{N_1, V_1} = \left. \frac{\partial \ln \Omega_m(\omega_m)}{\partial E_m} \right|_{N_m, V_m}$$

3) $\Rightarrow$

$$\begin{aligned} B_1 &= B_m \\ T_1 &= T_m \end{aligned} \quad \begin{aligned} M &= 2, 3, \dots, \infty \\ M &= 2, 3, \dots, \infty \end{aligned}$$

4) So, using same steps before

$$\frac{d \ln \Omega_m}{dS} = \frac{\left(\frac{\partial \ln \Omega_m}{\partial E_m} \right)_{N_m, V_m}}{\left(\frac{\partial S}{\partial E} \right)_{N_m, V_m}}$$

S/c

$$\frac{d \ln \Omega_m}{dS} = B, T_i = \left(\frac{1}{T_m} \right) = \left(\frac{1}{T_i} \right)$$

END PROOF

K constant = K

61) a) $du = \delta Q + \delta W$
 b) for "reversible heat transfer"
 δQ

Lee 30
 509.13

I) ARGUMENT / PROOF THAT AS $E \uparrow$, $S \uparrow$

At last time we proved that

1) $\Delta A = \sqrt{E} \sqrt{\Delta E} \frac{2\pi}{M}$ (ptcl. in a box)

2) AS $\Delta A \uparrow$ $\Delta \Omega \uparrow$ (shown/seen graphically)

3) AS system $E \uparrow$ $\Omega \uparrow$

4) For ΔA change in system E ,

$E_1 \rightarrow E_2$, $\Omega \rightarrow$ MAXIMUM POSSIBLE microstates (consistent w/ N, V, E_2)

i.e. we have a plausibility argument for postulate 2

5) FOR 1 ptcl. in an isothermal box, ~~the~~ an experimental measurement OF $[v_x, v_y]$ is likely

to lie anywhere w/in the Annulus $\Rightarrow$ plausibility arg for equal a priori probabilities



LEC 3
SQA 13 (3)

Implication ~~Conclusion~~ (Following from 1)-5):

$$\text{As } E \uparrow \Rightarrow dA \uparrow \Rightarrow R \uparrow \Rightarrow \text{from} \\ dS = k d \ln R \quad S \uparrow.$$

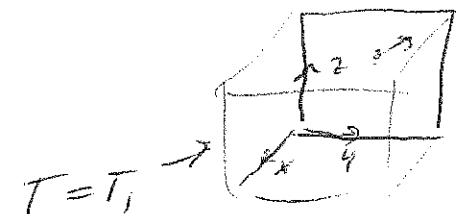
In other words, whenever we dump energy into a macrosystem's micro energy states, i.e., whenever we change the range of available molecular K.E., molecular rotational energy, molecular vibrational energy, molecular electronic energy, S increases

LEC 3
SP913

III) EXTENSION OF PLAUSIBILITY
ARGUMENT FOR FUNDAMENTAL
POSTULATE ($\Rightarrow$ EQUAL A PRIORI
PROBABILITIES OF MICROSTATES)
TO THE CASE OF N independent
ptcls

0) CONSIDER N independent ptcls in
~~AN~~ AN ISOTHERMAL BOX

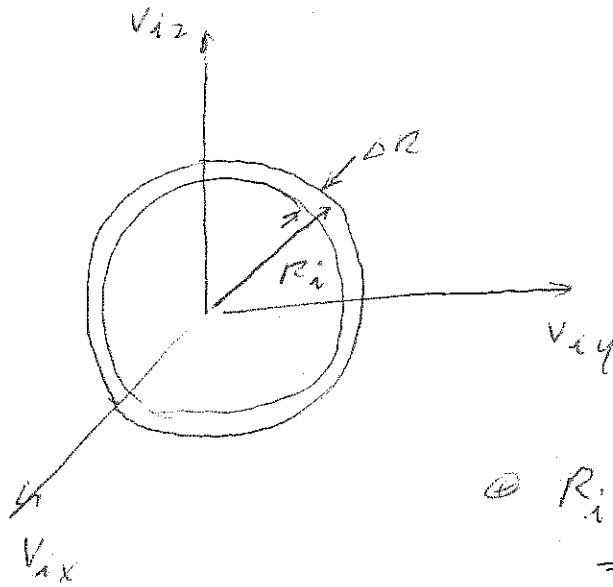
$$\vec{v}_i(t) = v_{ix}(t)\hat{e}_1 + v_{iy}(t)\hat{e}_2 + v_{iz}(t)\hat{e}_3$$
$$i = 1, 2, \dots, N$$



$T = T_i \rightarrow$
all
6 sides

1) SINCE EACH OF THE N PTCLS
ARE INDEPENDENT, THEN FOR
ANY ptcl, we have the same
picture as in Lec. 2!

Loc 8 (5)
 Sep 13



(F.9 5.1)

$$\oplus R_i = \sqrt{v_{ix}^2 + v_{iy}^2 + v_{iz}^2}$$

$$= \sqrt{\frac{2}{m} \bar{E}_i}$$

$$\oplus \Delta R = \sqrt{\frac{\Delta E}{m}}$$

$\oplus \Delta E$ fixed $\Leftrightarrow$ measurement
 uncertainty
 $\oplus$
 uncertainty in
 wall temp
 T_i

$\oplus \bar{E}_i$ is determined
 by $T_i = T_{wall}$ AND
 T_{measured} BY collisions
 w/ other
 particles

$\oplus$ NOTE; we are
~~the~~ ASSUMING
 that over
 a large number
 of measurements
 $\bar{E}_i$ is fixed;

specifically, assume:

Loc 3 (6)

Seq. B

$$\boxed{\bar{E}_i - \Delta E \leq E_i \leq \bar{E}_i + \Delta E} \quad (6.1)$$

~~and that~~

Assumed

where E_i is ^{kinetic energy} gaussian and where

ΔE defines the standard deviation or $\sim 68\%$ 'confidence interval',

i.e. $\sim 68\%$ of all measurements of ptcl i's ^{kinetic energy} will lie

satisfying to within 1 std. deviation, of the mean $\bar{E}_i$, ΔE_i

OBSERVATIONS/NOTES:

$$\begin{aligned} \text{Ques 1) } dV_i &= 4\pi R^2 dR \\ &= 4\pi \left(\frac{2}{m} \bar{E}_i \right) \sqrt{\frac{\Delta E}{m}} \end{aligned}$$

$$2) \therefore \text{AS } \bar{E}_i \uparrow \Rightarrow dV_i \uparrow \Rightarrow R_i \uparrow \quad \left[R_i = R_i(\bar{E}_i, N=1, \sigma_{\text{max}}) \right]$$

$i = 1, 2, \dots, N$

3) Assuming that at equilibrium,

$\bar{E}_i = E_0 = \text{CONSTANT}$, $i = 1, 2, \dots, N$, i.e., ALL N ptcls have the same mean kin. energy.

lec 3 ⑦

THEN IT IS NOW CLEAR (FROM

11 → 31) that EACH $\mathcal{R}_i$ ↑ AS $\bar{E}_i$ ↑

4) NOTE!
~~STATE~~

$$\mathcal{R}_{TOT}(N, \bar{E}_{TOT}, V_{BOX}) =$$

$$(7.1) \quad \mathcal{R}_1(1, E_0 \in \bar{E}_1, V_{BOX}) + \mathcal{R}_2(N=1, \bar{E}_2(E_0), V_{BOX}) + \dots + \mathcal{R}_N(N=L, \bar{E}_N=E_0, V_{BOX})$$

AND SINCE THERE'S NO PHYSICAL
REASON

5) SINCE THERE'S NO PHYSICAL
MECHANISM THAT WILL FORCE
INDEPENDENT MEASUREMENTS
OF any given pt's kinetic
energy into a particular
region of its "measurement
shell" (= Fig 5.1), then the
particle will be ~~from~~ uniformly
distributed w/in this shell.

lec. 3 (8)

6) LIKEWISE, IF WE HAD THE
ABILITY TO SIMULTANEOUSLY
MEASURE THE K.E.'S OF
ALL N PTCLS, OVER AND
OVER / REPEATEDLY, THESE
MEASUREMENTS WOULD BE
UNIFORMLY DISTRIBUTED OVER
THE N INDIVIDUAL "MEASUREMENT
SHELLS" ASSOCIATED W/ THE N PTCLS.

∴ 5) AND 6) ^{FOR THE CASE} ^{OF INDEPENDENT} PROVE THE PTCLS
VALIDITY OF THE FUNDAMENTAL
POSTULATE OF STAT THERMO
(= equal a priori probability
of observing a MACROSYSTEM
IN ANY OF ITS AVAILABLE
MICROSTATES)

7) NOTE! THE MODE OF ~~THE~~ N INDEPENDENT PARTICLES IN A MOTED BOX
 ^ CORRESPONDS TO A MODE OF AN IDEAL GAS ($\equiv$ NON-INTERACTING) GAS IN A MOTED BOX.

8) NOTE: THE ASSUMPTION OF GAUSSIAN PTCL. velocities (A M GAUSSIAN GAUSSIAN velocity distbn implies A GAUSSIAN K. ENERGY DISTBN -- MUST CONFIRM) IS FULLY CONSISTENT WITH THE MAXWELL Boltzmann picture of an ideal ~~gas~~ gas. We'll discuss latter later the course.

Loc. 3
SP912 (10)

- 10) From (7.1), Assuming/requiring that the number of (K. energy) microstates available to each indep. ptcl, $\Omega_1, \Omega_2, \Omega_3, \dots, \Omega_N$, are all identical ($\Omega_1 = \Omega_2 = \dots = \Omega_N$) (since each ptcl behaves as a single ptcl. in a box, having the fixed energy $\bar{E}_i = E_0$) From (7.1) leads to

$$(10.1) \quad \Omega_{\text{tot}} = \Omega_1^N (N=1, \bar{E}_1 = E_0, V = V_{\text{box}}) \\ = \Omega_1^N$$

- the behavior of
11) Since a swarm of N independent ptcls in a heated box is equivalent can be understood in terms of a single ptcl in a box, then using the same

Loc. 3

509.13(11)

From
argument above Loc. 2

~~Concerning the fact that~~
single the

(showing that the number of
microstates available to
a system goes to its
maximum possible value as the
system ~~approaches~~ equilibrium), we now
approaches

have a proof for the
same feature, ~~on the~~
applicable to the case of
 N indep. ptcls.