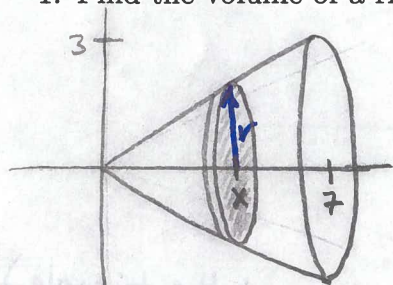


Examples

1. Find the volume of a right circular cone whose base radius is 3 and whose height is 7.

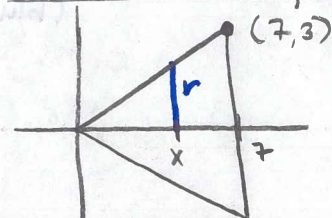


x varies from 0 to 7 so bounds will be 0 and 7

cross sections are circles (disks)

$\Rightarrow A(x) = \pi r^2$ where r depends on x

Flatten the picture:

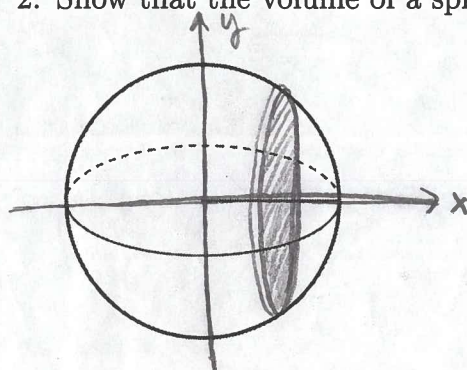


the line thru $(0,0)$ and $(7,3)$ is $y = \frac{3}{7}x$

so $r = \frac{3}{7}x$

$$\text{So } V = \int_0^7 A(x) dx = \int_0^7 \pi \left(\frac{3}{7}x\right)^2 dx = \frac{9\pi}{49} \left(\frac{1}{3}x^3\right) \Big|_0^7 = \frac{3\pi}{49} (7^3 - 0) = \boxed{21\pi}$$

2. Show that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

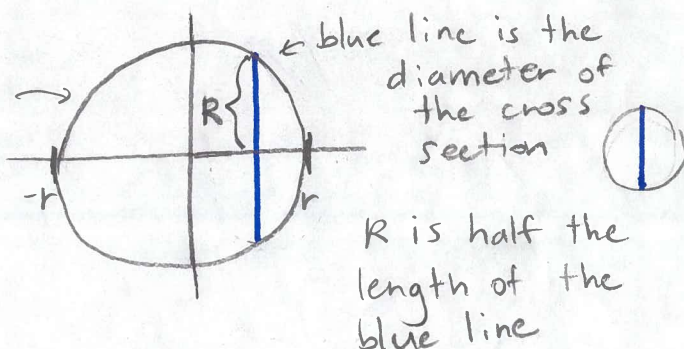


Again cross sections are circles

$A(x) = \pi R^2$, where R is a fun of x .

Flatten:

top half of circle has eq. $y = \sqrt{r^2 - x^2}$

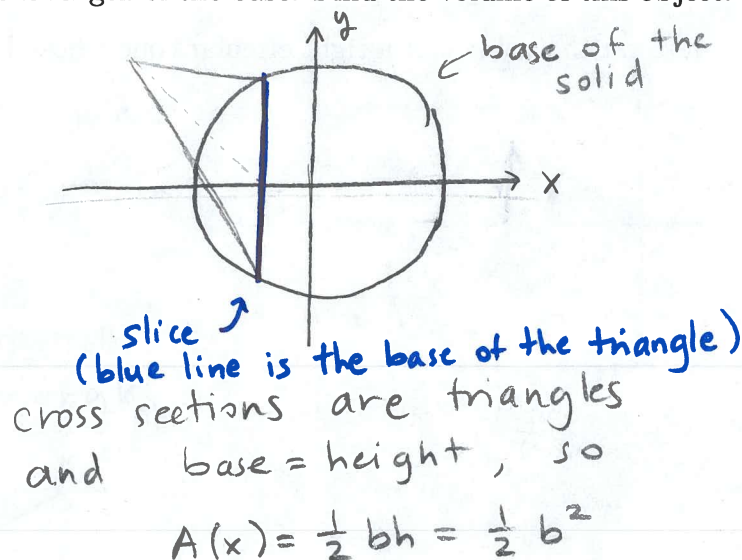
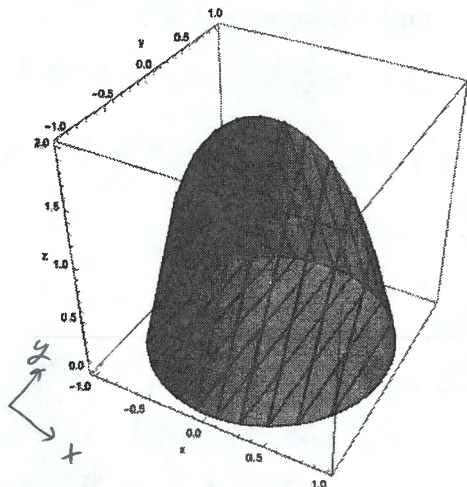


R is half the length of the blue line

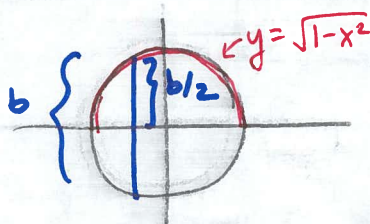
$$R = \sqrt{r^2 - x^2} \Rightarrow A(x) = \pi (\sqrt{r^2 - x^2})^2 = \pi (r^2 - x^2)$$

$$\begin{aligned} \text{So } V &= \int_{-r}^r \pi (r^2 - x^2) dx = \pi \int_{-r}^r r^2 - x^2 dx = \pi \left(r^2 x - \frac{1}{3} x^3 \right) \Big|_{-r}^r \\ &= \pi \left(\left(r^3 - \frac{1}{3} r^3 \right) - \left(-r^3 + \frac{1}{3} r^3 \right) \right) = \boxed{\frac{4}{3} \pi r^3} \end{aligned}$$

3. A solid object has as its base the circle $x^2 + y^2 = 1$. Each cross section perpendicular to the x -axis is a triangle whose height equals the length of the base. Find the volume of this object.



Want b in terms of x :



$$x^2 + y^2 = 1 \xrightarrow{\text{solve for } y} y = \pm \sqrt{1-x^2}$$

$$\left(\begin{array}{l} y = \sqrt{1-x^2} \text{ is top half} \\ \text{of the circle} \\ y = -\sqrt{1-x^2} \text{ is the} \\ \text{bottom half} \end{array} \right)$$

See from the picture that $\frac{b}{2} = \sqrt{1-x^2}$

$$\boxed{b = 2\sqrt{1-x^2}}$$

$$\text{So } A(x) = \frac{1}{2}b^2 = \frac{1}{2}(2\sqrt{1-x^2})^2 = 2(1-x^2)$$

$$\begin{aligned} \Rightarrow V &= \int_{-1}^1 A(x) dx = \int_{-1}^1 2(1-x^2) dx = 2 \left(x - \frac{1}{3}x^3 \right) \Big|_{-1}^1 \\ &= 2 \left(\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right) \right) \\ &= 2 \left(2 - \frac{2}{3} \right) = \left(\frac{8}{3} \right) \end{aligned}$$