8.5 Power Series

A **power series** is a series of the form

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \dots.$$

Here $\{c_n\}$ is the sequence of coefficients and a is the **center** or **base point** of the power series.

When we plug a value for x into a power series, we get a "normal" series.

Example: $\sum_{n=0}^{\infty} \frac{n}{2^n}(x-3)^n$

This is a power series with base point $a=3$ and coefficients $c_n = \frac{n}{2^n}$

When we plug in $x=2$, we get the series $\sum_{n=0}^{\infty} \frac{n}{2^n}(2-3)^n = \sum_{n=0}^{\infty}(-1)^n \frac{n}{2^n}$

A power series is a FUNCTION of x (plugging in a value for x gives exactly one output... EXCEPT when the series diverges, which is why we define the interval of convergence.)

Definition: The **interval of convergence** (IOC) of a power series is the set of all points, x, for which the power series converges.

Facts about the Interval of Convergence:

- If a power series converges at two separate points, it also converges at all points between them. In other words, the interval of convergence will always be an interval (unless it's a single point).
- The base point is always in the interval of convergence. Another way of saying this is that a power series always converges at its base point.
- The interval of convergence is always symmetric about the base point.

Definition: The **radius of convergence** of a power series $\sum c_n(x-a)^n$ is a number $R \geq 0$ such that

(i) if $|x-a| < R$, the series converges at x

(ii) if $|x-a| > R$, the series diverges at x

Facts about the Radius of Convergence:

- The radius of convergence does not tell you what happens when $|x-a| = R$.
- $R = \infty$ is allowed. If $R = \infty$, that means the power series converges at every real number, x, so the interval of convergence is $(-\infty, \infty)$.
- If $R = 0$, that means the power series converges only at the base point.
- We use the Ratio Test to find the radius of convergence.