

$$\textcircled{1} \int e^x \cos x \, dx = e^x \cos x + \int e^x \sin x \, dx = e^x \cos x + e^x \sin x - \int e^x \cos x \, dx$$

$$\boxed{\begin{array}{l} u = \cos x \quad v = e^x \\ du = -\sin x \, dx \quad dv = e^x \, dx \end{array}}$$

$$\boxed{\begin{array}{l} u = \sin x \quad v = e^x \\ du = \cos x \, dx \quad dv = e^x \, dx \end{array}}$$

$$\Rightarrow 2 \int e^x \cos x \, dx = e^x (\cos x + \sin x)$$

$$\Rightarrow \int e^x \cos x \, dx = \frac{1}{2} e^x (\cos x + \sin x) + C$$

$$\textcircled{2} \int x \ln x \, dx = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x \, dx$$

$$\boxed{\begin{array}{l} u = \ln x \quad v = \frac{1}{2} x^2 \\ du = \frac{1}{x} \, dx \quad dv = x \, dx \end{array}}$$

$$\boxed{= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C}$$

$$\textcircled{3} \int \cos^3 x \, dx = \int \cos^2 x \cdot \cos x \, dx$$

$$= \int (1 - \sin^2 x) \cos x \, dx$$

$$\boxed{u = \sin x \Rightarrow du = \cos x \, dx}$$

$$= \int 1 - u^2 \, du$$

$$= u - \frac{1}{3} u^3 + C$$

$$\boxed{= \sin x - \frac{1}{3} \sin^3 x + C}$$

$$\textcircled{4} \int \sec^3 x \, dx$$

$$= \sec(x) \tan(x) - \int \sec x \cdot \tan^2 x \, dx$$

$$\boxed{\begin{array}{l} u = \sec x \quad v = \tan x \\ du = \sec x \tan x \, dx \quad dv = \sec^2 x \, dx \end{array}}$$

$$= \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$$

$$= \sec(x) \tan(x) - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$\Rightarrow 2 \int \sec^3 x \, dx = \sec x \tan x + \ln |\sec x + \tan x|$$

$$\Rightarrow \int \sec^3 x \, dx = \frac{1}{2} [\sec(x) \tan(x) + \ln |\sec(x) + \tan(x)|] + C$$

$$\begin{aligned}
 \textcircled{5} \int \sin^3 x \cos^2 x \, dx &= \int \sin^2 x \cos^2 x \cdot \sin x \, dx \\
 &= \int (1 - \cos^2 x) \cos^2 x \sin x \, dx \quad \boxed{u = \cos x \Rightarrow du = -\sin x \, dx} \\
 &= \int (1 - u^2) u^2 (-du) \\
 &= \int u^4 - u^2 \, du \\
 &= \frac{1}{5} u^5 - \frac{1}{3} u^3 + C \quad \boxed{= \frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{6} \int_2^3 \frac{1}{x^2 - 1} \, dx \quad \frac{1}{x^2 - 1} &= \frac{A}{x-1} + \frac{B}{x+1} \Rightarrow 1 = A(x+1) + B(x-1) \\
 x=1 &\Rightarrow 1 = 2A \Rightarrow \boxed{A = 1/2} \\
 x=-1 &\Rightarrow 1 = -2B \Rightarrow \boxed{B = -1/2}
 \end{aligned}$$

$$\Rightarrow = \frac{1}{2} \int_2^3 \frac{1}{x-1} - \frac{1}{x+1} \, dx$$

$$\begin{aligned}
 &= \frac{1}{2} (\ln(x-1) - \ln(x+1)) \Big|_2^3 = \frac{1}{2} \ln\left(\frac{x-1}{x+1}\right) \Big|_2^3 \\
 &= \frac{1}{2} \left[\ln\left(\frac{2}{4}\right) - \ln\left(\frac{1}{3}\right) \right] \\
 &\quad \boxed{= \frac{1}{2} \left(\ln\left(\frac{3}{2}\right) \right)}
 \end{aligned}$$

$$\textcircled{7} \int \frac{10}{(x-1)(x^2+9)} dx$$

$$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$

$$\Rightarrow 10 = A(x^2+9) + (Bx+C)(x-1)$$

$$x=1 \Rightarrow 10 = 10A \Rightarrow \boxed{A=1}$$

$$x=0 \Rightarrow 10 = 9A - C \Rightarrow 10 = 9 - C \Rightarrow \boxed{C=-1}$$

$$x=-1 \Rightarrow 10 = 10A + (-B+C)(-2)$$

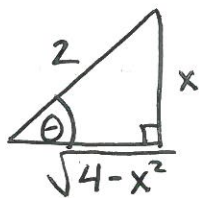
$$10 = 10 + 2B - 2C$$

$$0 = 2B + 2 \Rightarrow \boxed{B=-1}$$

$$= \int \frac{1}{x-1} - \frac{x}{x^2+9} - \frac{1}{x^2+9} dx$$

$$= \ln|x-1| - \frac{1}{2} \ln(x^2+9) - \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$$

$$\textcircled{8} \int \frac{dx}{x^2 \sqrt{4-x^2}}$$



$$\sin \theta = \frac{x}{2} \Rightarrow x = 2 \sin \theta$$

$$\Rightarrow dx = 2 \cos \theta d\theta$$

$$\sec \theta = \frac{2}{\sqrt{4-x^2}} \Rightarrow \frac{1}{\sqrt{4-x^2}} = \frac{\sec \theta}{2}$$

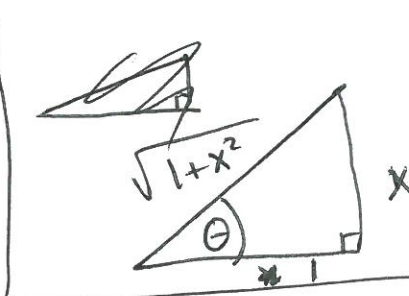
$$= \int \frac{1}{x^2} \cdot \frac{1}{\sqrt{4-x^2}} dx = \int \frac{1}{4 \sin^2 \theta} \cdot \frac{\sec \theta}{2} \cdot 2 \cos \theta d\theta$$

$$= \int \frac{1}{4} \csc^2 \theta d\theta$$

$$= -\frac{1}{4} \cot \theta + C$$

$$= -\frac{1}{4} \frac{\sqrt{4-x^2}}{x} + C$$

⑨ $\int \sqrt{1+x^2} dx$



$\tan \theta = x$
 $\sec^2 \theta d\theta = dx$
 $\sec \theta = \sqrt{1+x^2}$

$$= \int \sec \theta \cdot \sec^2 \theta d\theta = \int \sec^3 \theta d\theta$$

$$= \frac{1}{2} \left[\sec \theta \tan \theta + \ln |\sec \theta + \tan \theta| \right] + C \quad (\text{see problem 4})$$

$$= \frac{1}{2} \left[x\sqrt{1+x^2} + \ln |x + \sqrt{1+x^2}| \right] + C$$

⑩ $\int_0^1 \ln x dx = \lim_{t \rightarrow 0^+} \int_t^1 \ln(x) dx = \lim_{t \rightarrow 0^+} x \ln x - x \Big|_t^1$ (Integrate by parts)

$$= -1 - \left(\lim_{t \rightarrow 0^+} t \ln t - t \right)$$

$$= -1 - \left(\lim_{t \rightarrow 0^+} t \ln t \right)$$

$$\lim_{t \rightarrow 0^+} t \ln(t) = \lim_{t \rightarrow 0^+} \frac{\ln(t)}{1/t} = \lim_{t \rightarrow 0^+} \frac{1/t}{-1/t^2} = \lim_{t \rightarrow 0^+} -t = 0$$

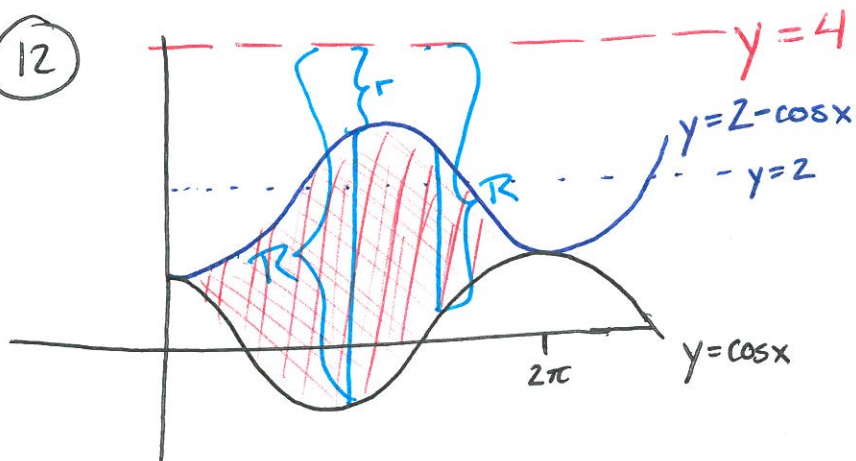
$$= -1$$

⑪ $\int_0^\infty x e^{-x} dx = -x e^{-x} \Big|_0^\infty + \int_0^\infty e^{-x} dx = 0 + e^{-x} \Big|_0^\infty$

$u = x \quad v = -e^{-x}$
 $du = dx \quad dv = e^{-x} dx$

$$= 1$$

12



$$V = \pi \int_0^{2\pi} R^2 - r^2 dx = \pi \int_0^{2\pi} (4 - \cos x)^2 - (4 - (2 - \cos x))^2 dx$$

$$= \pi \int_0^{2\pi} (4 - \cos x)^2 - (2 + \cos x)^2 dx$$

13

$$x = e^t \cos t$$

$$y = e^t \sin t$$

$$0 \leq t \leq \pi$$

$$\text{Arc Length} = \int_0^{\pi} \left[\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right]^{1/2} dt$$

$$\frac{dx}{dt} = e^t \cos t - e^t \sin t \Rightarrow \left(\frac{dx}{dt} \right)^2 = e^{2t} \cos^2 t - 2e^{2t} \sin t \cos t + e^{2t} \sin^2 t$$

$$\frac{dy}{dt} = e^t \sin t + e^t \cos t \Rightarrow \left(\frac{dy}{dt} \right)^2 = e^{2t} \sin^2 t + 2e^{2t} \sin t \cos t + e^{2t} \cos^2 t$$

$$\Rightarrow \left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 = e^{2t} (\cos^2 t + \sin^2 t) + e^{2t} (\sin^2 t + \cos^2 t) = 2e^{2t}$$

$$\Rightarrow \text{Arc Length} = \int_0^{\pi} \sqrt{2} e^t dt = \sqrt{2} e^t \Big|_0^{\pi}$$

$$= \sqrt{2} (e^{\pi} - 1)$$

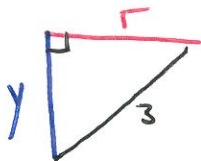
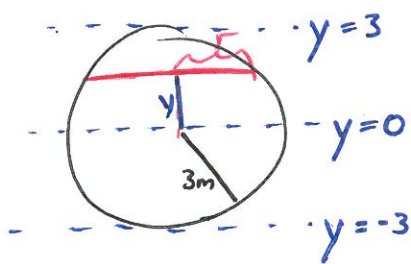
$$(14) f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

$$= \frac{1}{\pi} \int_0^{\pi} 2\sin(x) - \sin(2x) dx$$

$$= \frac{1}{\pi} \left[-2\cos(x) + \frac{1}{2}\cos(2x) \right] \Big|_0^{\pi}$$

$$= \frac{1}{\pi} \left[-2(-1-1) + \frac{1}{2}(1-1) \right] = \frac{4}{\pi}$$

(15)



$$y^2 + r^2 = 9$$

$$r^2 = 9 - y^2$$

$$\text{Work} = \int dw$$

$$= \int \text{distance} * dF$$

$$= \int (3-y) (pg\pi r^2 dy)$$

$$= \int_{-3}^3 (3-y) (1000 \cdot 9.8 \cdot \pi) (9-y^2) dy$$

$$= \text{something.}$$

$$\textcircled{16} \quad \frac{dy}{dx} = \frac{xy \sin(x)}{y+1} \quad y(0)=1 \quad \Rightarrow \quad \frac{y+1}{y} dy = x \sin(x) dx$$

$$\Rightarrow \int 1 + \frac{1}{y} dy = \int x \sin x dx = -x \cos x + \int \cos x dx$$

$$\boxed{\begin{array}{l} u=x \quad v=-\cos x \\ du=dx \quad dv=\sin x dx \end{array}}$$

$$\Rightarrow y + \ln|y| = \sin x - x \cos x + C$$

$$y(0)=1 \Rightarrow 1 + \ln(1) = \sin(0) - 0 \cdot \cos(0) + C$$

$$\Rightarrow 1 = C$$

$$\boxed{\Rightarrow y + \ln|y| = \sin x - x \cos x + 1}$$

$$\textcircled{17} \quad y' \cot^2 x = 1+y \quad y(\pi/3)=1$$

$$\Rightarrow \frac{dy}{1+y} = \tan^2 x dx \Rightarrow \ln|1+y| = \int \sec^2 x - 1 dx$$

$$= \tan x - x + C$$

$$\Rightarrow 1+y = C \cdot e^{\tan x - x}$$

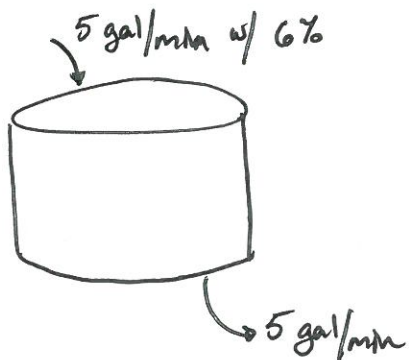
$$\Rightarrow y = C e^{\tan x - x} - 1$$

$$y(\pi/3)=1 \Rightarrow 1 = C e^{0 - \pi/3} - 1 \Rightarrow C = 2e^{\pi/3}$$

$$\boxed{y = 2e^{\tan x - x + \pi/3}}$$

(18)

500 gal

Let y = alcohol in tank (in gallons)

$$y' = (\text{rate}_{\text{in}}) - (\text{rate}_{\text{out}})$$

$$= 5 \cdot (0.06) - (5) \left(\frac{y}{500} \right)$$

$$= \frac{3}{10} - \frac{1}{100} y$$

$$\Rightarrow 100 \frac{dy}{dt} = 30 - y \quad \Rightarrow \frac{dy}{30-y} = \frac{1}{100} dt$$

$$\Rightarrow -\ln|30-y| = \frac{1}{100} t + C$$

$$\Rightarrow \ln|30-y| = -\frac{1}{100} t + C \Rightarrow 30-y = C e^{-\frac{1}{100} t}$$

$$\Rightarrow y = 30 + C e^{-\frac{1}{100} t}$$

When $t=0$, $y = 500 \cdot (0.06) = 30$

$$\Rightarrow 30 = 30 + C \quad \Rightarrow C = 0$$

$$\Rightarrow y = 30 + 0 e^{-\frac{1}{100} t}$$

When $t=60$, $y = 30 + 0 e^{-\frac{3}{5}}$

$$\Rightarrow \text{Tank has } \frac{30 + 0 e^{-3/5}}{500} \text{ alcohol by volume}$$

①9 Let y = temperature of the turkey at time t

$$\Rightarrow \frac{dy}{dt} = k(T_e - y) \quad \text{where } T_e = \text{temperature of the environment}$$

$$\Rightarrow \frac{dy}{dt} = k(75 - y) \Rightarrow \frac{dy}{75 - y} = k dt$$

$$\Rightarrow -\ln|75 - y| = kt + C$$

$$\Rightarrow 75 - y = e^{-kt} \cdot C$$

$$\Rightarrow y = 75 + Ce^{-kt}$$

$$\text{Since } y(0) = 180 \Rightarrow 180 = 75 + C \Rightarrow C = 105$$

$$\Rightarrow y = 75 + 105e^{-kt}$$

$$y(1/2) = 150 \Rightarrow 150 = 75 + 105e^{-k/2}$$

$$\Rightarrow \frac{75}{105} = e^{-k/2}$$

$$\Rightarrow \ln\left(\frac{75}{105}\right) = -\frac{k}{2} \Rightarrow -2 \cdot \ln\left(\frac{75}{105}\right) = k$$

$$\Rightarrow y = 75 + 105e^{2 \cdot \ln(75/105) t}$$

$$= 75 + 105\left(\frac{75}{105}\right)^{2t}$$

$$45 \text{ min} \Rightarrow t = 3/4 \Rightarrow \boxed{\text{temp} = 75 + 105\left(\frac{75}{105}\right)^{3/2}}$$

$$(20) \quad \frac{dP}{dt} = (5 \times 10^{-2})P - (5 \times 10^{-4})P^2$$

a) For logistic growth, $\frac{dP}{dt} = (kP)(1 - \frac{P}{M})$ where M = carrying capacity.

So if $\frac{dP}{dt} = 0$, we know $P=0$ or $P=M$

$$\Rightarrow (5 \times 10^{-2})P - (5 \times 10^{-4})P^2 = 0 \Rightarrow P(5 \times 10^{-2} - (5 \times 10^{-4})P) = 0$$

$$\text{If } P \neq 0, \quad P = \frac{5 \times 10^{-2}}{5 \times 10^{-4}} = \frac{1}{10^{-2}} = 100$$

b) $P(0) = 10$ means if $t=0$, $P=10$

$$\Rightarrow P'(0) = (5 \times 10^{-2})(10) - (5 \times 10^{-4})(10^2)$$

$$= 0.5 - 0.05$$

$$= 0.45$$

(21) $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n \cdot 3^n}$ Consider $a_n = \frac{(3x-2)^n}{n \cdot 3^n}$ and $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$

$$\lim_{n \rightarrow \infty} \left| \frac{(3x-2)^{n+1}}{(n+1) 3^{n+1}} \right| \cdot \left| \frac{n \cdot 3^n}{(3x-2)^n} \right| = \lim_{n \rightarrow \infty} |3x-2| \cdot \frac{n}{n+1} \cdot \frac{3^n}{3^{n+1}}$$

$$= \frac{1}{3} |3x-2| < 1$$

$$\Rightarrow |x - 2/3| < 1$$

We have absolute convergence when $-1 < x - 2/3 < 1$

$$\Rightarrow -1/3 < x < 5/3$$

For $x = -1/3 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1-2)^n}{n \cdot 3^n} = \sum_{n=1}^{\infty} \frac{(-3)^n}{n \cdot 3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges

$x = 5/3 \Rightarrow \sum_{n=1}^{\infty} \frac{(5-2)^n}{n \cdot 3^n} = \sum_{n=1}^{\infty} \frac{3^n}{n \cdot 3^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges

$$\Rightarrow \left[-1/3, 5/3 \right)$$

(22) $f(x) = \frac{x}{2x^2+1}$

$$\frac{1}{1+2x^2} = \frac{1}{1-(2x^2)} = \sum_{n=0}^{\infty} (-2x^2)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n \cdot 2^n \cdot x^{2n}$$

$$\Rightarrow \frac{x}{1+2x^2} = \sum_{n=0}^{\infty} (-1)^n \cdot 2^n \cdot x^{2n+1}$$

(23) $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$ is a geometric series that converges b/c $|\frac{1}{\sqrt{2}}| < 1$

(24) $\sum_{n=0}^{\infty} \arctan(n)$ diverges b/c $\lim_{n \rightarrow \infty} \arctan(n) = \frac{\pi}{2} \neq 0$

(25) $\sum_{n=0}^{\infty} \frac{n^3+1}{3n^4-1}$ Let $a_n = \frac{n^3+1}{3n^4-1} > 0$ and $b_n = \frac{1}{n}$

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{n^4+1}{3n^4-1} = \frac{1}{3} \neq 0. \text{ Since } \sum_{n=1}^{\infty} \frac{1}{n} \text{ diverges,}$$

$\sum_{n=0}^{\infty} \frac{n^3+1}{3n^4-1}$ diverges by limit comparison test

(26) $\sum_{n=1}^{\infty} \sin(\frac{1}{n})$ Since $0 < \frac{1}{n} < \frac{\pi}{2}$, $0 < \sin(\frac{1}{n}) < 1$

Let $a_n = \sin(\frac{1}{n})$, $b_n = \frac{1}{n}$

$$\lim_{n \rightarrow \infty} \frac{\sin(\frac{1}{n})}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\cos(\frac{1}{n}) \cdot -\frac{1}{n^2}}{-\frac{1}{n^2}} = \lim_{n \rightarrow \infty} \cos(\frac{1}{n}) = 1 \neq 0$$

Since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, $\sum_{n=1}^{\infty} \sin(\frac{1}{n})$ diverges by L.C.T.

$$(27) \sum_{n=0}^{\infty} \frac{2^n \cdot n^3}{n!} \quad \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \frac{2^{n+1} \cdot (n+1)^3}{(n+1)!} \cdot \frac{n!}{2^n \cdot (n)^3} \\ = \lim_{n \rightarrow \infty} 2 \cdot \left(\frac{n+1}{n}\right)^3 \cdot \frac{1}{n+1} = 0 < 1$$

So converges by ratio test

(28) $\sum_{n=0}^{\infty} \frac{\sqrt{n}}{1+n^2}$ $\frac{\sqrt{n}}{1+n^2} < \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}}$ $\sum_{n=0}^{\infty} \frac{1}{n^{3/2}}$ converges by p-test ($p > 1$)

Since $\frac{\sqrt{n}}{1+n^2} > 0$, $\sum_{n=0}^{\infty} \frac{\sqrt{n}}{1+n^2}$ converges by direct comparison.

$$(29) \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{4^n (2n)!} \quad \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

$$\text{If } x = \pi/2, \quad x^{2n} = \left(\frac{\pi}{2}\right)^{2n} = \frac{\pi^{2n}}{2^{2n}} = \frac{\pi^{2n}}{4^n}$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot \frac{\pi^{2n}}{4^n} = \cos\left(\frac{\pi}{2}\right) = 0$$

$$(30) \sum_{n=1}^{\infty} \frac{3^n}{n \cdot 5^n} \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \Rightarrow -\ln|1-x| = \sum_{n=0}^{\infty} \frac{1}{n+1} x^{n+1} + C$$

If $x=0$, we see $C=0$

$$\Rightarrow -\ln(1-x) = \sum_{n=0}^{\infty} \frac{1}{n+1} x^{n+1} \quad \text{for } |x| < 1$$

$$\text{Using an index shift } \Rightarrow = \sum_{n=1}^{\infty} \frac{1}{n} \cdot x^n$$

$$\Rightarrow -\ln(1 - 3/5) = \sum_{n=1}^{\infty} \frac{1}{n} \cdot \frac{3^n}{5^n}$$

$$\Rightarrow = -\ln(2/5) = \ln(5/2)$$

$$(31) \sum_{n=1}^{\infty} \frac{3^n}{5^n \cdot n!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n = 1 + x + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 + \dots$$

$$\Rightarrow e^x - 1 = \sum_{n=1}^{\infty} \frac{1}{n!} x^n$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n!} \frac{3^n}{5^n} = e^{3/5} - 1$$

$$(32) \begin{cases} x = \cos \theta + \sin 2\theta \\ y = \sin \theta + \cos 2\theta \end{cases} @ \theta = \pi/3$$

$$\text{Slope} = \frac{dy}{dx} \Big|_{\theta=\pi/3} = \frac{(dy/d\theta)}{(dx/d\theta)} \Big|_{\theta=\pi/3}$$

$$\frac{dy}{d\theta} = \cos \theta - 2 \sin(2\theta), \text{ when } \theta = \pi/3 \Rightarrow = \frac{1}{2} - 2 \frac{\sqrt{3}}{2} = \frac{1}{2} - \sqrt{3}$$

$$\frac{dx}{d\theta} = -\sin \theta + 2 \cos(2\theta), \text{ when } \theta = \pi/3 \Rightarrow = -\frac{\sqrt{3}}{2} + 2(-\frac{1}{2}) = -\frac{\sqrt{3}}{2} - 1$$

$$\Rightarrow m = \frac{\frac{1}{2} - \sqrt{3}}{-\frac{\sqrt{3}}{2} - 1} = \frac{2\sqrt{3} - 1}{-\sqrt{3} - 2}$$

$$m = \frac{\frac{1}{2} - \sqrt{3}}{-1 - \sqrt{3}} = \frac{\sqrt{3} - 1/2}{\sqrt{3} + 1} = \frac{2\sqrt{3} - 1}{2\sqrt{3} + 2}$$

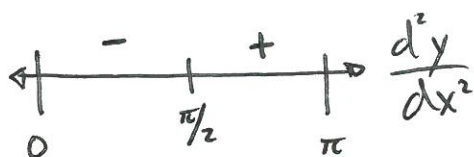
$$(33) \begin{cases} x = \cos 2t \\ y = \cos t \end{cases} 0 < t < \pi$$

$$\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{-\sin(t)}{-2\sin(2t)} = \frac{\sin(t)}{4\sin(t)\cos(t)}$$

$$= \frac{1}{4} \sec(t)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \cdot \frac{1}{dx/dt} = \frac{1}{4} \sec(t) \cdot \tan(t) \cdot \frac{1}{-4\sin(t)\cos(t)}$$

$$= \frac{-1}{16 \cos^3(t)}$$



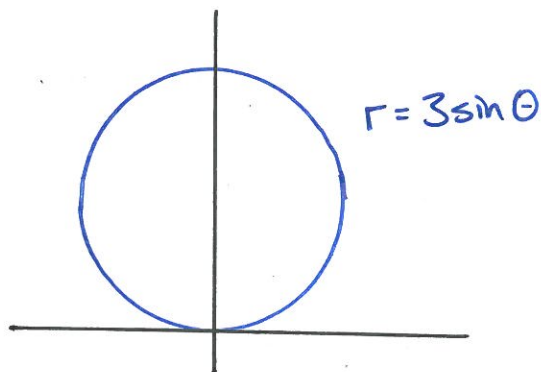
So concave up on $(\pi/2, \pi)$

(34) If $r = 3 \sin \theta$, $\Rightarrow r^2 = 3r \sin \theta \Rightarrow x^2 + y^2 = 3y$

$$\Rightarrow x^2 + y^2 - 3y = 0 \Rightarrow x^2 + \left(y - \frac{3}{2}\right)^2 - \frac{9}{4} = 0$$

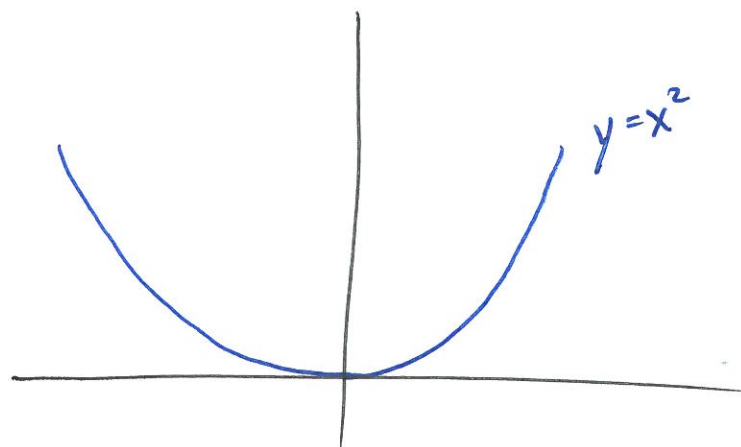
$$\Rightarrow x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$$

So a circle of $r = \frac{3}{2}$, centered at $(0, \frac{3}{2})$.



(35) If $r = \tan \theta \sec \theta$, $\Rightarrow r \cos \theta = \tan \theta$

$$\Rightarrow x = \frac{y}{x} \Rightarrow x^2 = y$$



(36) $r = \cos(2\theta)$ when $\theta = \pi/4$

Since $x = r \cos \theta \Rightarrow \frac{dx}{d\theta} = r' \cos \theta - r \sin \theta$

$y = r \sin \theta \Rightarrow \frac{dy}{d\theta} = r' \sin \theta + r \cos \theta$

$\Rightarrow \left. \frac{dy}{d\theta} \right|_{\theta=\pi/4} = [-2 \sin(2\theta)] \sin \theta + \cos(2\theta) \cos(\theta) \Big|_{\theta=\pi/4}$

$= -2 \sin(\pi/2) \sin(\pi/4) + \cos(\pi/2) \cos(\pi/4)$

$= -2 \cdot \frac{1}{\sqrt{2}} = -\sqrt{2}$

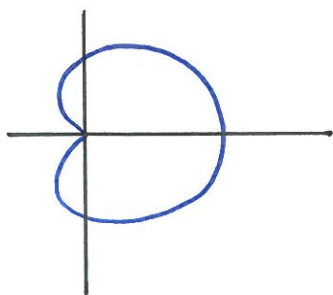
$\left. \frac{dx}{d\theta} \right|_{\theta=\pi/4} = [-2 \sin(2\theta)] \cos \theta - \cos(2\theta) \sin(\theta) \Big|_{\theta=\pi/4}$

$= -2 \sin(\pi/2) \cos(\pi/4) - \cos(\pi/2) \sin(\pi/4)$

$= -2 \cdot \frac{1}{\sqrt{2}} = -\sqrt{2}$

$\frac{dy}{dx} = \text{slope} = \frac{dy/d\theta}{dx/d\theta} \Rightarrow \boxed{\text{slope} = 1}$

(37) $r = 1 + \cos \theta$



Area $= \int_0^{2\pi} \frac{1}{2} r^2 d\theta$

$= 2 \cdot \int_0^{\pi} \frac{1}{2} (1 + \cos \theta)^2 d\theta$

$= \int_0^{\pi} 1 + 2\cos \theta + \cos^2 \theta d\theta$

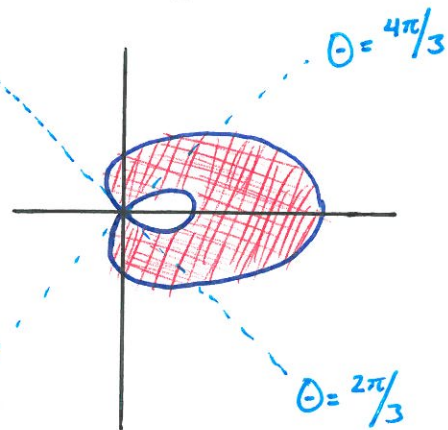
$= \int_0^{\pi} \frac{3}{2} + 2\cos \theta + \frac{1}{2} \cos 2\theta d\theta$

$= \left. \frac{3}{2} \theta + 2\sin(\theta) + \frac{1}{4} \sin(2\theta) \right|_0^{\pi}$

$= \boxed{\frac{3}{2} \pi}$

(38)

$$r = \frac{1}{2} + \cos \theta$$

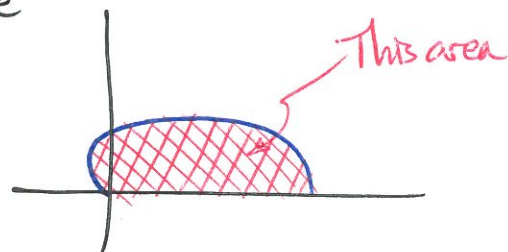


$$\text{Area} = \int \frac{1}{2} r^2 d\theta$$

If we integrate

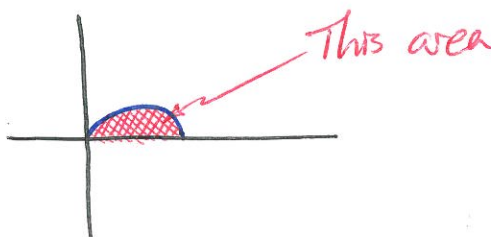
$$\int_0^{2\pi/3} \frac{1}{2} \left[\frac{1}{2} + \cos \theta \right]^2 d\theta$$

we compute



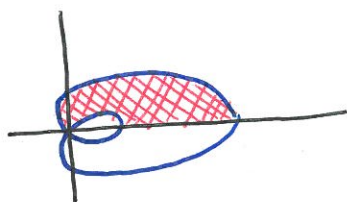
$$\text{So } \int_0^{2\pi/3} \frac{1}{2} \left[\frac{1}{4} + \cos \theta + \cos^2 \theta \right] d\theta = \frac{3\sqrt{3} + 4\pi}{16} \quad (\text{yeah I skipped some stuff})$$

If we integrate $\int_{\pi}^{4\pi/3} \frac{1}{2} \left[\frac{1}{2} + \cos \theta \right]^2 d\theta$, we compute



$$\text{So } \int_{\pi}^{4\pi/3} \frac{1}{2} \left[\frac{1}{4} + \cos \theta + \cos^2 \theta \right] d\theta = \frac{2\pi - 3\sqrt{3}}{16}$$

Thus, $\frac{4\pi + 3\sqrt{3}}{16} - \frac{2\pi - 3\sqrt{3}}{16} = \frac{\pi + 3\sqrt{3}}{8}$ is this area:



The region is symmetric about $y=0$ ($\theta=0$), so the total is

$$\boxed{\frac{\pi + 3\sqrt{3}}{4}}$$

$$(39) \quad r = 3 \sin \theta \quad 0 \leq \theta \leq \pi/3 \quad \Rightarrow \quad r' = 3 \cos \theta$$

$$\begin{aligned} \text{Arc Length} &= \int_0^{\pi/3} [r^2 + (r')^2]^{1/2} d\theta \\ &= \int_0^{\pi/3} [9 \sin^2 \theta + 9 \cos^2 \theta]^{1/2} d\theta \\ &= \int_0^{\pi/3} 3 d\theta = 3\theta \Big|_0^{\pi/3} = \boxed{\pi} \end{aligned}$$

$$(40) \quad r = \theta^2 \quad 0 \leq \theta \leq 2\pi \quad \Rightarrow \quad r' = 2\theta$$

$$\begin{aligned} \text{Arc Length} &= \int_0^{2\pi} [\theta^4 + 4\theta^2]^{1/2} d\theta \\ &= \int_0^{2\pi} |\theta| \sqrt{\theta^2 + 4} d\theta \end{aligned}$$

Since $0 \leq \theta \leq 2\pi$, $|\theta| = \theta$

$$\Rightarrow = \int_0^{2\pi} \theta \sqrt{\theta^2 + 4} d\theta$$

$$\begin{aligned} u = \theta^2 + 4 &\Rightarrow du = 2\theta d\theta & \theta = 0 &\Rightarrow u = 4 \\ \theta = 2\pi &\Rightarrow u = 4(\pi+1) \end{aligned}$$

$$\begin{aligned} \text{Arc length} &\Rightarrow \int_4^{4(\pi+1)} \sqrt{u} du = \frac{2}{3} u^{3/2} \Big|_4^{4\pi+1} \\ &= \boxed{\frac{2}{3} [(4\pi+1)^{3/2} - 8]} \end{aligned}$$