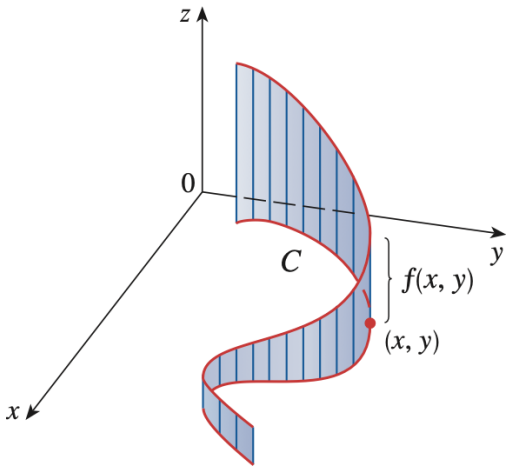
Line Integrals of Scalar Fields

Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ and C is a smooth curve in the xy-plane.

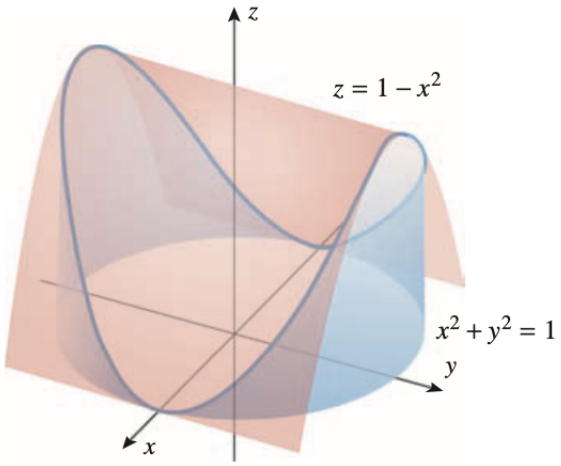


If C is parameterized by $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ for $a \leq t \leq b$, then

$$\int_C f(x,y)\, ds = \int_a^b f(\mathbf{r}(t))\, \|\mathbf{r}'(t)\|\, dt$$

$$= \int_a^b f(x(t), y(t)) \sqrt{x'(t)^2 + y'(t)^2}\, dt.$$

1. The surface area of the part of the cylinder $x^2 + y^2 = 1$ that lies between the xy-plane and $z = 1 - x^2$ can be expressed as a line integral. If you were to express this line integral, what function would you be integrating and what curve C would you be integrating over?

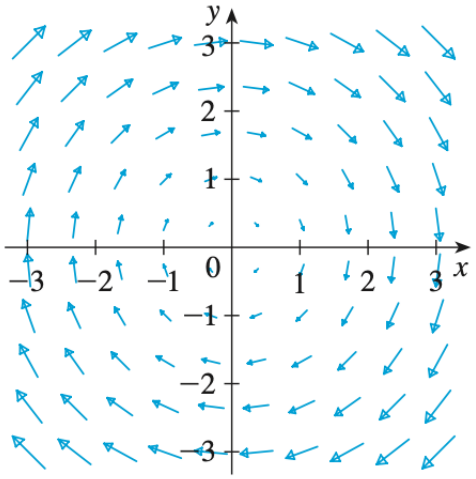


Line Integrals of Vector Fields

Suppose $\mathbf{F}$ is a vector field and C is a smooth, oriented curve in the domain of $\mathbf{F}$ that is parametrized by $\mathbf{r}(t)$ where $a \leq t \leq b$. Then the line integral of $\mathbf{F}$ along C is

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\, dt.$$

2. Let $\mathbf{F}$ be the vector field shown in the figure.



(a) If C_1 is the vertical line segment from $(3, -3)$ to $(3, 3)$, determine whether $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$ is positive, negative or zero.

(b) If C_2 is the circle of radius 3 centered at the origin, oriented counterclockwise, determine whether $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ is positive, negative or zero.

(c) If C_3 is the horizontal line segment from $(-3, 0)$ to $(3, 0)$, determine whether $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$ is positive, negative or zero.