Practice Problems - The Invertible Matrix Theorem

1. Determine if each matrix is invertible. Give as many different justifications as you can think of while doing as few computations as possible.

(a) $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 3 & 1 \\ 0 & 4 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 0 & 7 \\ 3 & 0 & -1 \\ 2 & 0 & 9 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

(f) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$

(g) $\begin{bmatrix} 1 & 2 & -3 \\ 1 & -3 & 2 \\ -3 & 2 & 1 \end{bmatrix}$

(h) $\begin{bmatrix} 1 & 4 & 2 \\ 3 & 0 & 3 \\ 1 & 4 & 2 \end{bmatrix}$

2. Can a square matrix with two identical columns be invertible? two identical rows?

3. The matrix $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$ is *diagonal* since the only nonzero entries are on the main diagonal.

   (a) Find D^{-1}.

   (b) When is an $n \times n$ diagonal matrix invertible?

   (c) Explain how to find the inverse of a diagonal matrix in general. (What is the relationship between the diagonal entries of D and D^{-1}?)

4. (a) If B is a square matrix and the equation $B\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ in $\mathbb{R}^n$, what can you say about the equation $B\mathbf{x} = \mathbf{0}$? About the matrix B?

   (b) If B is a square matrix and the equation $B\mathbf{x} = \mathbf{c}$ is consistent for all $\mathbf{c}$ in $\mathbb{R}^n$, what can you say about the equation $B\mathbf{x} = \mathbf{0}$? About the matrix B?

   (c) Answer part (a) and (b) if B is not square.

5. Suppose A is a 4×4 matrix such that $A\mathbf{x} = \mathbf{e}_i$ is consistent for each i between 1 and 4. Is A invertible?

6. Suppose A is a 3×3 matrix such that $A\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = \mathbf{0}$. Do the columns of A span $\mathbb{R}^3$?

7. Suppose A is an invertible matrix and $AB = AC$. Can you conclude that $B = C$?

8. True or False? If A is invertible, then A^2 is invertible.

9. True or False? If A^2 is invertible, then A is invertible.