

Secure and Efficient Flexibility Service Procurement: A Game-Theoretic Approach

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Overview

- 1 Introduction
- 2 Market model and game formulation
- 3 Market clearing protocol
- 4 Case study
- 5 Summary

Background

- Increasing penetration of uncertain and volatile **renewable energy generation** into power system
- Increasing need for **flexibility** (adjustment of energy generation or consumption)
- **Energy consumers** providing flexibility for balancing with DERs



Centralized Power Plants → Distributed Energy Resources (DERs)

Goal & Challenges

Goal: Ensuring an **efficient** and **secure** operation of the balancing/flexibility market

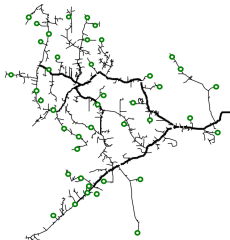


- Self-interested behaviors of energy consumers
- Distribution network operation constraints
- Limited information sharing

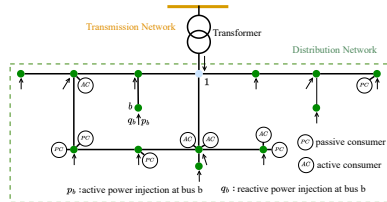
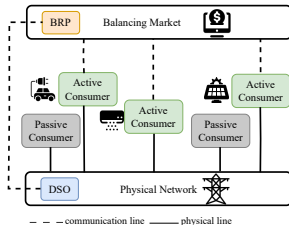
Prisoners' dilemma

	prisoner B	
	confess	remain silent
prisoner A	 5 years 5 years	 0 year 20 years
	 20 years 0 year	 1 year 1 year

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Balancing market model



- **BRP(market operator)**: procures flexibility services for balancing
- **Energy consumers**: provide flexibility by adjusting their DERs.
- **DSO(system operator)**: clears the market subject to physical network constraints.

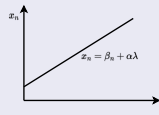
Balancing market model

Energy consumer perspective

Bidding: Affine function bidding

$$x_n = \beta_n + \alpha\lambda.$$

x_n : flexibility, λ : price, β_n : decision variable



Economic interest:

$$\begin{aligned} \min J_n(x_n, \lambda) = & \underbrace{C_n(x_n)}_{\text{cost of providing flexibility}} - \underbrace{\lambda x_n}_{\text{payment from BRP}} \\ \text{s.t.} \quad & \underbrace{0 \leq x_n \leq \hat{x}_n}_{\text{local flexibility constraint}} \end{aligned}$$

Market perspective

■ Required total flexibility: $\sum_{n \in \mathcal{N}} x_n = x_{\text{tot}}$

■ Physical network constraints:

Linear power flow:

$$P_{\mathcal{L}} = -WE\theta + UEv, \quad Q_{\mathcal{L}} = -UE\theta - WEv,$$

$$E^{\top} P_{\mathcal{L}} = P_{\mathcal{B}}(\mathbf{x}), \quad E^{\top} Q_{\mathcal{L}} = Q_{\mathcal{B}},$$

Line capacity constraints:

$$P_{\mathcal{L}} \odot P_{\mathcal{L}} + Q_{\mathcal{L}} \odot Q_{\mathcal{L}} \leq Z \odot Z,$$

Voltage and angle limitations:

$$\underline{\theta} \mathbf{1} \leq \theta \leq \bar{\theta} \mathbf{1}, \quad \underline{v} \mathbf{1} \leq v \leq \bar{v} \mathbf{1}.$$

$P_{\mathcal{L}}$: active power flow
 $Q_{\mathcal{L}}$: reactive power flow
 $P_{\mathcal{B}}$: active power injection
 $Q_{\mathcal{B}}$: reactive power injection
 θ : voltage angle
 v : voltage magnitude
 U : conductance
 W : susceptance
 E : incidence matrix
 Z : maximum line capacity
 $\odot$: Hadamard product

Game formulation

Bids: $x_n = \beta_n + \alpha\lambda$ Flexibility requirement: $\sum_{n \in \mathcal{N}} x_n = x_{\text{tot}}$

$\Downarrow$

Clearing Price: $\lambda = (x_{\text{tot}} - \mathbf{1}^\top \beta) / (\alpha N)$

Allocated flexibility: $x_n = (x_{\text{tot}} - \mathbf{1}^\top \beta) / N + \beta_n$

$$\begin{aligned} \min_{\beta_n} J_n(\beta_n, \beta_{-n}) &= \underbrace{C_n((x_{\text{tot}} - \mathbf{1}^\top \beta) / N + \beta_n)}_{C_n(x_n)} \\ &\quad - \underbrace{(x_{\text{tot}} - \mathbf{1}^\top \beta + N\beta_n)(x_{\text{tot}} - \mathbf{1}^\top \beta) / (\alpha N^2)}_{\lambda x_n} \end{aligned}$$

Game among energy consumers:

$$\mathcal{G} = \{\mathcal{N}, K, \text{col}(J_n(\beta_n, \beta_{-n}))_{n \in \mathcal{N}}\},$$

where $K = \{\beta \mid x(\beta) \in \Theta, \Theta = \Theta_{\text{local}} \cap \Theta_{\text{physical}}\}$.

Game properties

Assumption

The cost function $C_n(x_n)$ is convex and twice continuously differentiable.
 $C'_n := \nabla C_n$ is Lipschitz continuous with $\kappa > 0$

Lemma

The game $\mathcal{G}$ has a unique v-GNE if $\alpha < \frac{2}{\kappa(N-1)}$, given by the unique solution $\beta^* \in K$ of the variational inequality $\text{VI}(K, F)$, i.e.

$$(\beta - \beta^*)^\top F(\beta^*) \geq 0, \quad \forall \beta \in K.$$

Here F is the pseudo-gradient mapping of the game $\mathcal{G}$,

$$F := \text{col}(f_n(\beta_n, \beta_{-n}))_{n \in \mathcal{N}},$$

where

$$\begin{aligned} f_n(\beta_n, \beta_{-n}) &:= \frac{\partial}{\partial \beta_n} J_n(\beta_n, \beta_{-n}) \\ &= C'_n(x_n) \frac{N-1}{N} + \frac{(\mathbf{1}^\top \beta - x_{\text{tot}})(N-2) + N\beta_n}{\alpha N^2}. \end{aligned}$$

Efficiency analysis

The social welfare optimization problem:

$$\min_{x \in \Theta} \sum_{n \in \mathcal{N}} C_n(x_n) \quad \text{s.t.} \quad \sum_{n \in \mathcal{N}} x_n = x_{\text{tot}}$$

Lemma

Let $x^* = \text{col}(x_n^*)_{n \in \mathcal{N}}$ be the allocated flexibility at v-GNE, namely $x_n^* = (x_{\text{tot}} - \mathbf{1}^\top \beta^*)/N + \beta_n^*$, then x^* is the optimal solution of the following minimization problem,

$$\min_{x \in \Theta} \sum_{n \in \mathcal{N}} D_n(x_n) \quad \text{s.t.} \quad \sum_{n \in \mathcal{N}} x_n = x_{\text{tot}}$$

with $D_n(x_n) = C_n(x_n) + \frac{x_n^2}{2\alpha(N-1)}$.

Efficiency analysis

PoA (Price of Anarchy): The ratio between the total cost at Nash Equilibrium and the total cost at the social optimum.

$$\text{PoA} := \frac{\sum_{n \in \mathcal{N}} C_n(x_n^*)}{\sum_{n \in \mathcal{N}} C_n(\bar{x}_n)}$$

Proposition

Let $\bar{x}$ be the social optimum and x^* be the allocated flexibility at v -GNE of the game $\mathcal{G}$. Then, we have

$$\text{PoA} = \frac{\sum_{n \in \mathcal{N}} C_n(x_n^*)}{\sum_{n \in \mathcal{N}} C_n(\bar{x}_n)} < 1 + \frac{1}{2\alpha(N-1)} \frac{\sum_{n \in \mathcal{N}} \bar{x}_n^2}{\sum_{n \in \mathcal{N}} C_n(\bar{x}_n)}$$

Market clearing protocol

Objectives

Economic interest of the consumers ✓

Local flexibility constraints ✓

Physical network constraints ✓

All captured by:

$$\beta \longrightarrow \beta^* \in K \text{ (v-GNE)}, \quad x \longrightarrow x(\beta^*) \in \Theta_{local} \cap \Theta_{physical}$$

Respecting information sharing limitations

BRP: the total flexibility mismatch x_{tot} .

Energy consumers: the economic and technical characteristics such as the cost function $C_n(x_n)$ and the maximum flexibility $\hat{x}_n$.

DSO: the system configurations such as network topology and line parameters.

- **Energy consumer:** updates its primary bid as

$$\beta'_n(k+1) = \beta_n(k) - \rho_n f_n(\beta_n(k), \beta_{-n}(k))$$

with $f_n(\beta_n(k), \beta_{-n}(k)) = h_n(\lambda(k), \beta_n(k), \gamma_n(k), \mathbf{1}^\top \gamma(k))$.

- **DSO:** adjusts the bids by solving the following optimization problem,

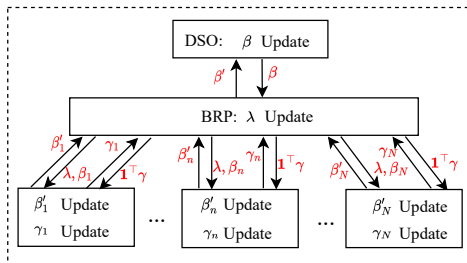
$$\beta(k+1) = \operatorname{argmin}_{z \in \Theta_p} \|z - \beta'(k+1)\|$$

- **BRP:** clears the market price $\lambda(k+1) = (x_{\text{tot}} - \mathbf{1}^\top \beta(k+1)) / (\alpha N)$.

- **Energy consumer:** updates γ_n as

$$\gamma_n(k+1) = \operatorname{proj}_{\mathbb{R}^+}(\gamma_n(k) + v_n(2x_n(k+1) - x_n(k) - \hat{x}_n))$$

with $x_n(k+1) = \beta_n(k+1) + \alpha \lambda(k+1)$.



Convergence analysis

The algorithm in a compact form:

$$\begin{aligned}\beta(k+1) &= \text{proj}_{\Theta_{\text{physical}}}(\beta(k) - \rho(F(\beta(k)) + A^\top \gamma(k))) \\ \gamma(k+1) &= \text{proj}_{\mathbb{R}_N^+}(\gamma(k) + v(2A\beta(k+1) - A\beta(k) - \hat{x} + b))\end{aligned}$$

Proposition

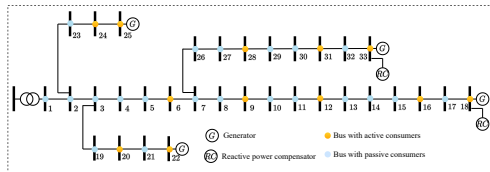
Let $\bar{\rho} := \max\{\rho_n\}_{n \in \mathcal{N}}$ and $\bar{v} := \max\{v_n\}_{n \in \mathcal{N}}$, if

$$\frac{\kappa_F^2}{2\eta_F} < \frac{1}{\bar{\rho}} - \bar{v},$$

Then, the sequence generated by the algorithm converges to $\text{col}(\beta^*, \gamma^*)$ with β^* the v -GNE of game $\mathcal{G}$, where $\eta_F = \frac{1}{\alpha N} - \frac{\kappa(N-1)}{2N}$ and $\kappa_F = \frac{N-1}{N}(\kappa + \frac{1}{\alpha})$.

Case study

IEEE 33 distribution network:



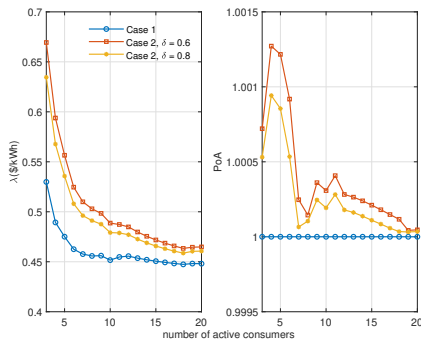
simulation setting

Cost functions: $C_n(x_n) = \frac{1}{2}a_n x_n^2 + b_n x_n$, with $a_n \in [0.003, 0.005] \text{ \$/ (kWh)}^2$ and $b_n \in [0.35, 0.45] \text{ \$/kWh}$.

The total flexibility requirement is $x_{\text{tot}} = 100 \text{ kWh}$.

The BRP chooses $\alpha = \delta \frac{2}{\kappa(N-1)}$ with $\delta \in (0, 1)$.

Efficiency analysis

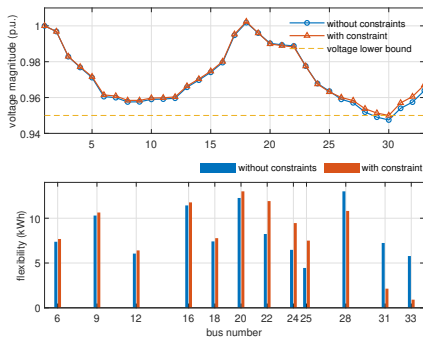


Case 1: social welfare optimization problem.

Case 2: proposed mechanism.

Security analysis

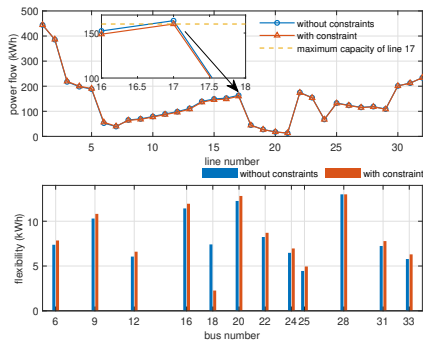
Scenario 1: There is an energy supply surplus.



Consumer 31 and 33 consume more energy to compensate the extra energy supply, which leads to the voltage violation at bus 30.

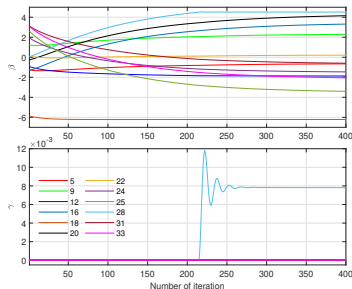
Security analysis

Scenario 2: There is an energy supply deficit.

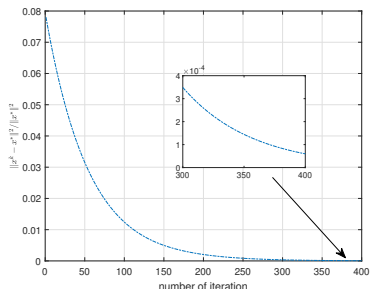


Consumer 18 generates more energy to compensate the insufficient energy supply, which leads to the congestion at line 17.

Convergence analysis



The evolution of bids β and dual variables γ



The normalized errors of flexibility profile x

$$\|x^k - x^*\|^2 / \|x^*\|^2$$

Scalability analysis

Table: The convergence rate

Number of consumers	10	20	30	40
Number of iterations	215	459	518	693

The required number of iterations increases linearly with respect to the number of active consumers

Summary

Conclusion

- A market mechanism for procuring flexibility services from energy consumers in distribution network.
- The market mechanism is efficient since the Generalized Nash Equilibrium of the game is close to the social optimum.
- The market mechanism is secure since the physical network constraints are incorporated.
- The algorithm can find the Generalized Nash Equilibrium with limited information sharing.

Future work

- Incorporating uncertainties into the model of consumers.
- Beyond affine bidding functions.