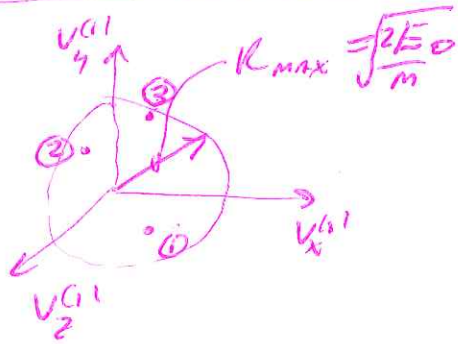


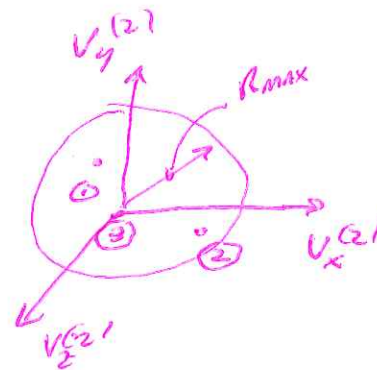
EXAMPLE 2 - PLAUSIBILITY OF EQUAL

SYSTEM = 2 PARTICLES IN A HEATED BOX

A PRIORI PROBABILITIES



ptcl 1



ptcl 2

NOTES:

- 1) $E_0 = E_1 + E_2 = \frac{1}{2} m (v_x^{(1)2} + v_y^{(1)2} + v_z^{(1)2}) + \dots$
 $\left\{ + \frac{1}{2} m (v_x^{(2)2} + v_y^{(2)2} + v_z^{(2)2}) \right\}$
- 2) $N=2$
- 3) OBSERVATIONS (1), (2) & (3) OF INDIVIDUAL PARTICLE ENERGIES SHOWN
- 4) THE MAX. POSSIBLE ENERGY OF EITHER PARTICLE IS E_0 ; THE MINIMUM IS 0. THUS, IN VELOCITY-SPACE, WE CAN OBSERVE EITHER PARTICLE W/in THE SPHERES SHOWN ABOVE.
- 5) INTUITIVELY (NOT RIGOROUSLY).

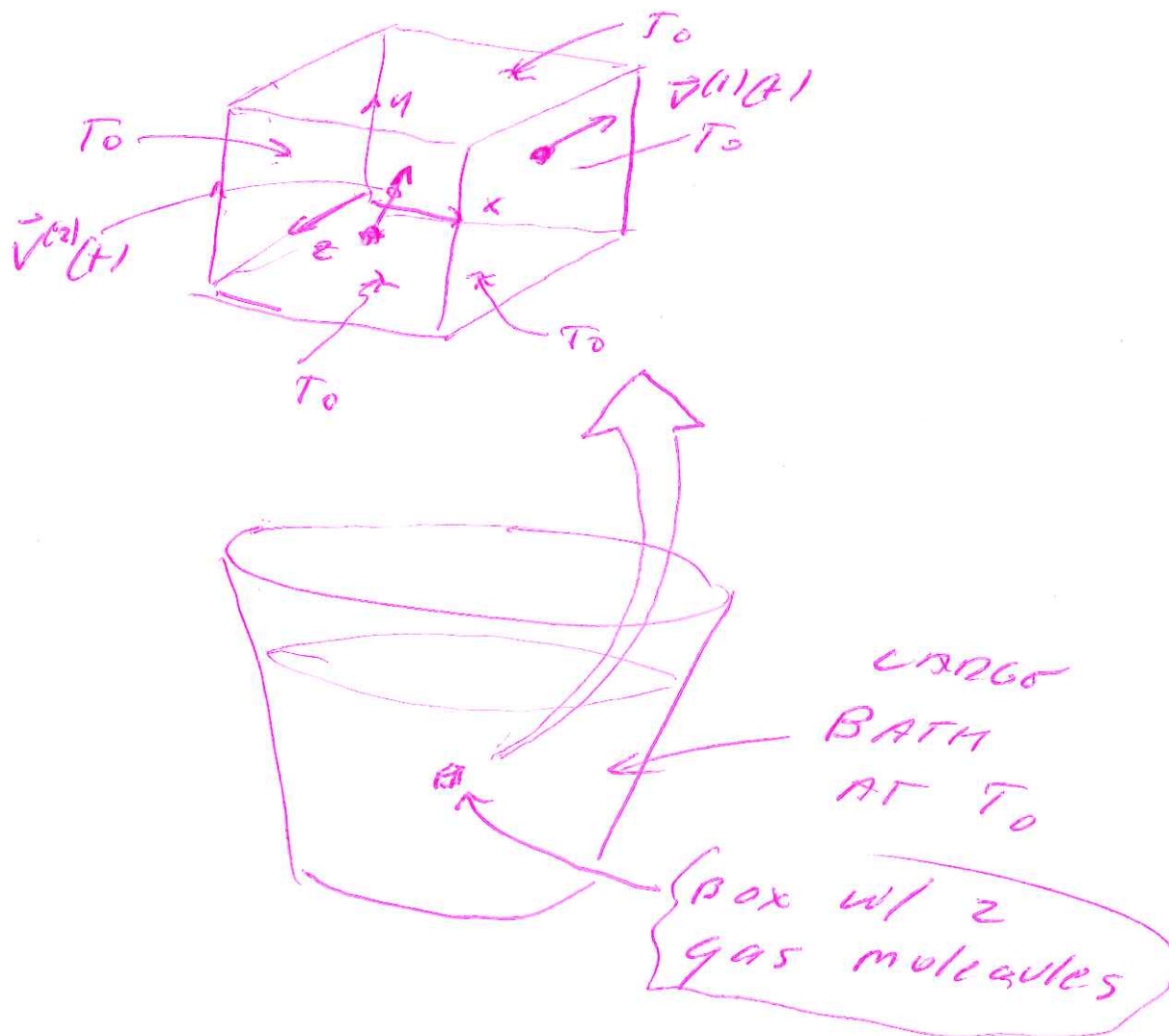
IT SEEMS REASONABLE THAT
 THE PROBABILITY OF OBSERVING

THE 2 particles IS UNIFORM

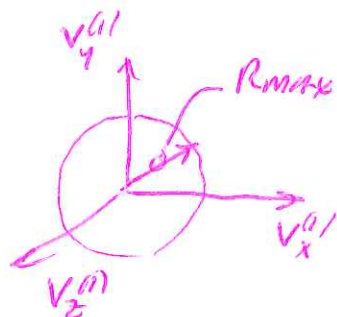
W/IN THE STATE SPACE
SYSTEM

(= THE 2 SPHERES IN V-SPACE).

6) PICTURE OF PHYSICAL SYSTEM;

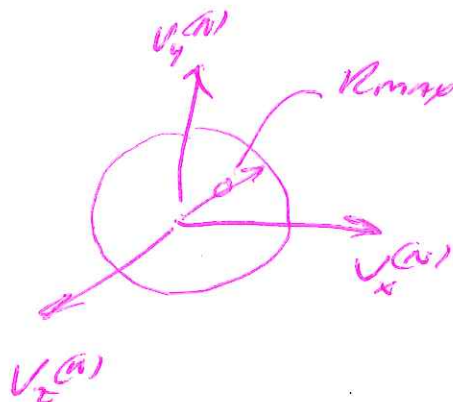


7) FINALLY FOR N
INDEPENDENT (IDEAL GAS)
PARTICLES, WE CAN
 SKETCH N SPHERES: ($N \approx 10^{23}$)



ptcl. 1

...



ptcl N

NOTES:

a) $E_0 = \frac{1}{2} m \sum_{a=1}^N \underline{v^{(a)}} \cdot \underline{v^{(a)}}$

b) MAX POSSIBLE ^(BUT EXTREMELY IMPROBABLE) ENERGY FOR
 ANY GIVEN PARTICLE = E_0

$$\therefore R_{max} = \sqrt{\underline{v^{(a)}} \cdot \underline{v^{(a)}}}_{max}$$

$$= \sqrt{\frac{2E_0}{m}}$$

c) MIN POSSIBLE ENERGY FOR
 ANY ptcl = 0.

$\therefore$ THUS, WE GOT THE
 ABOVE SKETCH.

(4)

8) QUICK ESTIMATE OF PROBABILITY
OF OBSERVING 1 OF $N (= O(10^{23}))$
PTCLS HAVING TOTAL SYSTEM
ENERGY E_0

a) GIVEN A ^{VELOCITY} MEASUREMENT
UNCERTAINTY OR Δv , LETS
USE THIS TO FIRST CALCULATE
THE NUMBER OF v COLLS OR
MEASUREMENT
BINS, N_c , EXIST W/IN
ANY ONE OF THE N
SPHERES (HAVING MAX $R = \sqrt{\frac{2}{m} E_0}$)

$$i) E_{\text{max measured 1 ptcl}} = \frac{1}{2} m (v_x^2 + v_y^2 + v_z^2) + \frac{1}{2} m 2 \underline{v} \cdot \underline{\Delta v}$$

$$\uparrow$$

FROM $E_{\text{max}} = \frac{1}{2} m [(v_x + \Delta v)^2 + (v_y + \Delta v)^2 + (v_z + \Delta v)^2]$

$$\text{OR } \frac{2 E_{\text{max 1 ptcl}}}{m} = \left(\frac{\underline{v} \cdot \underline{v}}{v^2} \right) + 2 \underline{v} \cdot \underline{\Delta v} \quad (= N_{\text{1 ptcl}}^{\text{max}})$$

$$\text{OR } R_{\text{1 ptcl}}^{\text{max}} = \sqrt{v^2 \left[1 + \frac{2 \underline{v} \cdot \underline{\Delta v}}{v^2} \right]}$$

$$= v \left[1 + 2 \frac{\Delta v}{v} \right]^{1/2} \quad \left(\begin{array}{l} \text{choose, eg.,} \\ \underline{v} = v_x \hat{e}_x \end{array} \right)$$

OR $\Delta R_{\text{inter}} = V \left[1 + \frac{1}{2} \left(\frac{2\Delta V}{V} \right) \right]$ ←

Taylor expand RHS of last line of pg ④

OR $R_{\text{inter}}^{\text{max}} = \left(\overset{\uparrow}{R} \right) + \left(\overset{\uparrow}{\Delta R} \right)$
 $R_{\text{inter}}^{\text{max}} = \left(\overset{\uparrow}{V} \right) + \left(\overset{\uparrow}{\Delta V} \right)$ (*)

where again $V = |\vec{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2}$.

∴ (*) shows ^{ISOTOPIE} that we can divide each of the N spheres (in vel-space) into N_c cells/bins, where

$$N_c \approx \frac{\frac{4}{3}\pi R_{\text{max}}^3}{\Delta R^3}$$

$$\boxed{N_c \approx \frac{\frac{4}{3}\pi \left(\frac{2E_0}{M} \right)^{3/2}}{(\Delta V)^3}} \quad (5.1)$$

$E_0 = \underline{\text{TOTAL ENERGY}}$ OF N
 GAS molecules w/in
 ISOTHERMAL BOX

$M =$ MASS of each molecule
 $\Delta V =$ (known) velocity measurement uncertainty

④ (5.1) is AN approximate value for N_c since we're discretizing each sphere w/ cubes

(6)

b) Given N and N_c , then

the ^(approx) probability of observing

ALL OR THE SYSTEM ENERGY,

E_0 , residing in a single

ptcl. (during any random/
and purely hypothetical and simultaneous
observation of all N

ptcls = $\prod$ - PROBABILITY
of observing a ^{single} ptcl.

w/in the outer shell of a
single sphere (w/in the set of
 N spheres) $\prod$

i) volume of outer shell

$$= 4\pi R_{\max} \Delta R = \Delta V_{\text{outer}}$$

$$= 4\pi \sqrt{\frac{2}{m E_0}} \Delta V$$

ii) Number of cells, N_{surf} ,

w/in this shell =

$$N_{\text{surf}} \approx \frac{\Delta V_{\text{outer}}}{\Delta V}$$

$$\approx \frac{4\pi R_{\max}^2 \Delta R}{\Delta R^3}$$

or

$$N_{\text{surf}} \approx \frac{4\pi \left(\frac{2E_0}{m}\right)^{3/2}}{\Delta V^2} \quad (9.1)$$

iii) $\therefore$ probability of observing a given ptcl. w/in the outer-most shell, $P_{\text{outer}}|_{\text{ptcl}}$, is

THUS :

$$\begin{aligned} P_{\text{outer}}|_{\text{ptcl}} &\approx \frac{N_{\text{surf}}}{N_c} \\ &\approx \frac{4\pi \left(\frac{2E_0}{m}\right)^{3/2} / \Delta V^2}{\frac{4}{3}\pi \left(\frac{2E_0}{m}\right)^{3/2} / \Delta V^2} \end{aligned}$$

$$(9.2) \quad P_{\text{outer}}|_{\text{ptcl}} \approx \frac{3 \Delta V}{V_{\text{max}}} = \frac{3 \Delta V}{\sqrt{\frac{2E_0}{m}}}$$

iv) Since there are N spheres and N associated outer shells AND since we're measuring the kinetic energy of all N ptcls simultaneously, then the prob. of observing 1 ptcl in one of N shells is :

$$\frac{N \cdot N_{\text{surf}}}{N \cdot N_c} = \frac{N_{\text{surf}}}{N_c}$$

(8)

V) Finishing details:

$$1) E_0 = N \langle E \rangle$$

$\langle E \rangle$ = avg energy for a

$$2) \langle E \rangle = \frac{3}{2} k T_0 \quad (\text{equipartition of energy})$$

single ptcl

$$3) \therefore \boxed{E_0 = \frac{3}{2} N k T_0}$$

ass'd)

within the box

(more later)

k = Boltzmann's constant

4) Let's assume

$$\Delta v / \langle v \rangle \approx 10^{-3}$$

$$k = 1.38 (10^{-23}) \frac{\text{J}}{\text{K}}$$

$$T_0 = 300 \text{ K}$$

where

$$M \approx \frac{30 \text{ kg/kmol}}{6 (10^{26}) / \text{kmol}} \approx 5 (10^{-26}) \text{ kg}$$

$$\langle E \rangle = \frac{1}{2} m \langle v^2 \rangle$$

$$\Rightarrow \sqrt{\langle v^2 \rangle} = \sqrt{\frac{\langle E \rangle 2}{m}} = \sqrt{\frac{(\frac{3}{2} k T_0) 2}{m}}$$

$$= \sqrt{\frac{3 k T_0}{m}}$$

$$\approx \sqrt{\frac{3 (1.38 (10^{-23}) \text{ J/K} \cdot 300 \text{ K})}{5 (10^{-26}) \text{ kg}}}$$

$$\approx 5 (10^2) \text{ m/s}$$

(seems ok - should

$$\Rightarrow \Delta v \approx 10^{-3} (500) \text{ m/s} \approx \underline{0.5 \text{ m/s}}$$

be on order of

$$5) \therefore \text{PMT } E_0 = N \langle E \rangle$$

340 m/s)

$$= N \frac{1}{2} m \langle v^2 \rangle$$

$$\approx$$

(9)

$$\begin{aligned}
 6) \therefore P &\approx \frac{3 \Delta V}{\sqrt{\frac{2 \epsilon_0}{m}}} = \frac{3 \Delta V}{\left(\frac{2}{m} N^{\frac{1}{2}} \hbar \langle v^2 \rangle \right)^{\frac{1}{2}}} \\
 &= \frac{3 (\Delta V / \sqrt{\langle v^2 \rangle})}{N^{\frac{1}{2}}} \\
 &= \underline{\underline{O(N^{-1/2}) \ll 1}}
 \end{aligned}$$