

Practice Problems - The Invertible Matrix Theorem

1. Determine if each matrix is invertible. Do as few computations as possible.

(a) $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 3 & 1 \\ 6 & 4 & 2 \end{bmatrix}$

(f) $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 3 & -5 \\ 3 & 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 0 & 7 \\ 3 & 0 & -1 \\ 2 & 0 & 9 \end{bmatrix}$

(g) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$

(h) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

2. An *upper triangular matrix* is a matrix where all entries below the main diagonal are zero. When is a square upper triangular matrix invertible?

3. Can a square matrix with two identical columns be invertible?
4. True or false? If the columns of A do not span $\mathbb{R}^n$, then the columns of A are linearly dependent.
5. **Theorem 10:** (*Using A^{-1} to solve a linear system.*)
If A is an $n \times n$ invertible matrix then for every $\mathbf{b} \in \mathbb{R}^n$, $A\mathbf{x} = \mathbf{b}$ has a unique solution given by $\mathbf{x} = A^{-1}\mathbf{b}$.
Why is this solution guaranteed to be unique?
6. Let A be a 3×3 matrix with the property that the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^3$ onto $\mathbb{R}^3$. Explain why the transformation must be one-to-one.
7. Suppose that the equation $A\mathbf{x} = \mathbf{b}$ has more than one solution for some $\mathbf{b}$ in $\mathbb{R}^n$. Choose the correct statement and justify why it is true.
- The transformation $\mathbf{x} \mapsto A\mathbf{x}$ is NOT one-to-one.
 - The transformation $\mathbf{x} \mapsto A\mathbf{x}$ is NOT onto.