

Lec. 4 spg. 17
 rough notes
 (version 2)

- 1) state $\Rightarrow \psi(x, t)$
- 2) a) ψ state given by
 a) $\langle \psi | \psi \rangle = P(x, t) =$ prob density of observing a system pre/ @ (x, t)
 2) a) $\Rightarrow \langle A \rangle, \text{var}(A) = \langle (A - \langle A \rangle)^2 \rangle$

$\langle A \rangle =$ avg over all quantum states

where a) $\langle A \rangle = \int \psi^* (\hat{A} \psi) dx$ from QM

a) postulate that

and b) observed exp.

$\langle A \rangle \Rightarrow \frac{\sum A_i}{N}$

ON $\hat{A}$

3) A observable from exp.

$P(x, t) = \psi^*(x, t) \psi(x, t)$

QM theory

$\Rightarrow \int \psi^*(x, t) \hat{A} \psi(x, t) dx$

$\hat{A} =$ operator

$=$ Math entity that extracts something from ψ by operating

$\underline{P}$	$\hat{P} = -i\hbar \nabla$	}	only become defined once into existence do something with req op. on ψ
$\underline{x}$	$\hat{x} = x$		
$\underline{E}$ total energy	$\hat{H} = \frac{\hat{P}^2}{2m} + \hat{V}(x, t)$		
$\underline{V(x)}$	$\hat{V} =$		

operators ~~are~~ these are stated as 2
 (4) 2 postulates (Kronecker - pg 180)
 a) to every observable

corresponds a linear
operator $\hat{A}$

b) $\langle A \rangle$ gotten by integration
 (summing $\psi^*(\hat{A}\psi)$ over all
 ind. vbls.

4) 3) a) S.E. $\leftarrow$ $\begin{cases} \oplus \text{ gov eqn for } \psi \\ \oplus \text{ merges ptcl and wave prop. of ptcls/systems} \end{cases}$
 $\hat{H}\psi = i\hbar\psi_t$

$\oplus \hat{H} =$

$\hbar = \frac{h}{2\pi}$

S.E. can be derived Heuristically
 using the 2 fundamental

5a) postulates of QM $\equiv$ PEderB

ptcl. prop. $\downarrow$

$\underline{p} = \hbar \underline{k} \Rightarrow$

$E = \hbar \omega$

wave like prop $\downarrow$

$\Rightarrow$

momentum of a quantum object is proportional to its wave number

" total Energy $= KE + PE$ prop. to its freq.

$$d_c = dv = \frac{2\pi}{1c} \frac{\omega}{2\pi} = \frac{\omega}{1c}$$

CD's prepared
for
FBI's
trial
by

—

wave packet
containing
a
no.
Four. comp

(1209) Idea 1 - Assume essential physics of dynamical system can be described via waves superposed

1. single Fourier comp.

$$a) \psi_n = A_n e^{i(k_n x - \omega_n t)}$$

⊕ PEderB relns

$$1) E_n = \hbar \omega_n$$

$$2) P_n = \hbar k_n$$

wave props

⊕ ptct props:

$$3) E_n = \frac{P_n^2}{2m}$$

(energy of n^{th} Fourier mode = $\hbar \omega_n$)

⊕ combine (1) & (3):

$$4) \frac{P_n^2}{2m} = \hbar \omega_n$$

⊕ use 2) in 4):

$$\cancel{\frac{\hbar^2 \omega_n^2}{2m} = \hbar k_n}$$

$$5) \boxed{\frac{\hbar^2 k_n^2}{2m} = \hbar \omega_n}$$

⊕ this has the form of a dispersion relation
 $\omega = \omega(k)$, which arises in wave problems

(key idea 2) capture both wave and ptcl props

⑤

5) provide a crucial hint
 at next step $\Rightarrow$ operate
 on the wave mode
 in 0) in such a way
 that you obtain the
 dispn. reln in 5)

6) 6a) $\frac{\partial^2}{\partial x^2} \psi_n = -k_n^2 \psi_n$

6b) $\frac{\partial}{\partial t} \psi_n = -i\omega_n \psi_n$

7) $\therefore$ mult 6a) by $\frac{\hbar^2}{2m}$ and 6b)

by $i\hbar \Rightarrow$

$$\boxed{\frac{\hbar^2}{2m} \psi_{n,xx} = i\hbar \psi_{n,t}}$$

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$\hat{p}^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}$$

= S.E.Q.W For free free
 motion of single
 ptcl. of mass m

6

SAME BUT NOW

$$E = \frac{1}{2m} p^2 + V(k, t)$$

WAVE PROPS

$$\Rightarrow p \in \mathbb{R}$$

$$E = \hbar \omega$$

$$p = \hbar k$$

WAVE PROPS

Assume

Ass 100% @ F.M.

$$\frac{1}{2m} p^2 + V = \hbar \omega$$

$$\boxed{\frac{\hbar^2 k^2}{2m} + V = \hbar \omega}$$

$$\Rightarrow \hbar \frac{\partial^2}{\partial x^2} \psi = -k^2 \psi$$

$$\frac{\partial}{\partial t} \psi = -i \omega \psi$$

What do we have to do to ψ to get this? $\psi \rightarrow \psi = 1$

$$\Rightarrow \ominus \frac{\hbar^2}{2m} \psi_{xx} + V \psi = i \hbar \psi_t$$

$$\vec{p} = -i \hbar \nabla$$

$$\vec{p}^2 = -\hbar^2 \nabla^2$$

$$\Rightarrow \boxed{\hat{H} \psi = i \hbar \psi_t}$$