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Office Hrs: Mon/Fri 11 - noon

on Zoom (same pass code)

Web assign - 15%

written HW - 15%

exams - 70%

Exams on Tues. 7-10pm

takes one hour (start at 9pm at latest)

First exam, next Tuesday

Use Proctorio (practice assignment)

Get on WebAssign today!

All HW due on Sundays!

Derivative practice : take the derivative

①  $x^7$   $7x^6$

⑤  $\sin x \cdot \cos x$

$$\cos x \cdot \cos x + \sin x \cdot (-\sin x) \\ = \cos^2 x - \sin^2 x$$

②  $\sqrt{x} = x^{1/2}$

$$\frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$$

⑥  $\sin(\cos x)$

$$\cos(\cos x) \cdot (-\sin x) \\ = -\cos(\cos x) \sin x$$

③  $\sqrt[3]{x^2} = x^{2/3}$

$$\frac{2}{3} x^{-1/3}$$

⑦  $x^3 e^{2x}$

$$e^{2x} \cdot 2 = 2e^{2x}$$

$$\text{answer: } 3x^2 \cdot e^{2x} + 2e^{2x} \cdot x^3$$

④  $\frac{5}{x} = 5x^{-1}$

$$5 \cdot (-x^{-2}) \\ = -\frac{5}{x^2}$$

⑧  $5^x$

similar to  $e^x$

$$\frac{d}{dx}(e^x) = e^x \cdot \underbrace{\ln(e)}_1$$

$$\text{answer: } \frac{d}{dx}(5^x) = 5^x \cdot \underbrace{\ln(5)}_{\text{constant}}$$

Why is  $\frac{d}{dx}(5^x) = 5^x \ln 5$ ?

$$y = 5^x$$

$$\ln y = \ln(5^x)$$

← log law

$$\ln y = x \ln 5$$

$$\frac{d}{dx}(\ln y) \overset{\text{fcn of } x}{=} \frac{d}{dx}(x \underline{\ln 5}) = \ln 5$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \ln 5$$



$$\frac{dy}{dx} = \ln 5 \cdot y = \ln 5 \cdot 5^x$$

## Integral/Antiderivative Practice:

$$\textcircled{1} \int x^2 dx$$

$$\frac{1}{3} x^3 + C$$

$$\textcircled{5} \int 5^x dx$$

$$= \frac{5^x}{\ln 5} + C$$

$$\frac{d}{dx} \left( \frac{5^x}{\ln 5} \right) = \frac{1}{\ln 5} \cdot (5^x \cdot \ln 5) = 5^x \quad \checkmark$$

$$\textcircled{2} \int \frac{1}{4} x^3 dx = \frac{1}{4} \int x^3 dx \quad \textcircled{6} \int \sec^2 x dx$$

$$\frac{1}{4} \cdot \left( \frac{1}{4} x^4 \right) + C$$

$$= \frac{1}{16} x^4 + C$$

$$= \tan x + C$$

$$\textcircled{3} \int \frac{1}{x} dx = \int x^{-1} dx$$

$$\ln |x| + C$$

$$\textcircled{7} \int \frac{1}{1+t^2} dt$$

$$= \arctan x + C$$

$$\text{or} \quad \tan^{-1} x + C$$

$$\textcircled{4} \int \sin \theta d\theta$$

$$- \cos \theta + C$$

$$\textcircled{8} \int_{-\pi/2}^{\pi/2} \cos x dx$$

$$= \sin x \Big|_{-\pi/2}^{\pi/2}$$

$$= \sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)$$

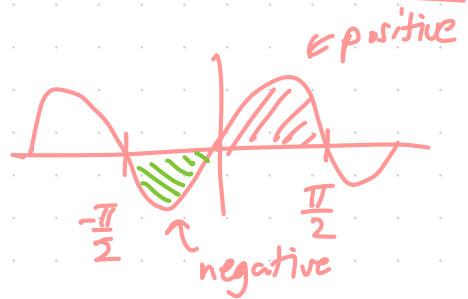
$$= 1 - (-1) = 2$$

$\cos x$  is an even function



$$\int_{-\pi/2}^{\pi/2} \cos x \, dx = 2 \int_0^{\pi/2} \cos x \, dx$$

$\sin x$  is an odd function



$$\int_{-\pi/2}^{\pi/2} \sin x \, dx = 0.$$

examples:

$$\textcircled{1} \int e^{5x} dx = \frac{1}{5} e^{5x} + C$$

check:  $\frac{1}{5} \cdot (5e^{5x}) \checkmark$

$$\textcircled{2} \int \sin\left(\frac{2}{3}x\right) dx = -\cos\left(\frac{2}{3}x\right) \cdot \frac{3}{2} + C$$

↑  
to counteract  
the  $\frac{2}{3}$   
that comes  
from chain rule

$$\begin{aligned} \text{check: } \frac{d}{dx} \left( -\frac{3}{2} \cos\left(\underline{\frac{2}{3}x}\right) \right) & \quad \leftarrow \text{chain rule.} \\ &= -\frac{3}{2} \cdot (-\sin\left(\frac{2}{3}x\right)) \cdot \frac{2}{3} \\ &= \sin\left(\frac{2}{3}x\right) \checkmark \end{aligned}$$

$$\textcircled{3} \int \sqrt{z} \cdot (z^2 + 6z + 4) dz$$

BIG MISTAKE: integral of a product is  
NOT the product of  
the individual  
integrals

$$\int \sqrt{z} (z^2 + 6z + 4) dz$$

$$\neq \int \sqrt{z} dz \cdot \int (z^2 + 6z + 4) dz$$

$$\int z^{1/2} (z^2 + 6z + 4) dz$$

$$= \int z^{5/2} + 6z^{3/2} + 4z^{1/2} dz$$

$$= \frac{2}{7} z^{7/2} + 6 \cdot \frac{2}{5} z^{5/2} + 4 \cdot \frac{2}{3} z^{3/2} + C$$

$$(4) \int \frac{6y^8 + 12y^2 - y^6}{3y^7} dy$$

strategy: simplify first!

← finish yourself

$$= \int \frac{6y^8}{3y^7} + \frac{12y^2}{3y^7} - \frac{y^6}{3y^7} dy$$

because sum is in numerator.

## u-substitution

Look for "function-derivative" pairs

e.g.  $\ln x \quad \& \quad \frac{1}{x}$   
 $x^3 \quad \& \quad x^2$   
 $\sqrt{x} \quad \& \quad \frac{1}{\sqrt{x}}$

← ignore constants

examples:

$$\textcircled{1} \int \frac{(\ln x)^2}{x} dx$$

function:  $\ln x$   
derivative:  $\frac{1}{x}$

$$\boxed{\begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array}}$$

$$\begin{aligned} &= \int (\ln x)^2 \cdot \underbrace{\frac{1}{x} dx}_{du} = \int u^2 du = \frac{1}{3} u^3 + C \\ &= \frac{1}{3} (\ln x)^3 + C \end{aligned}$$

$$(2) \int_{x=2}^{x=3} \frac{1}{3} x e^{x^2} dx$$

function:  $x^2$

deriv.:  $x$

$$u = x^2$$

$$du = 2x dx \rightarrow \left( \frac{1}{2} du \right) = x dx$$

$$= \int_{u=4}^{u=9} \frac{1}{3} \cdot e^u \cdot \frac{1}{2} du$$

$$= \frac{1}{6} e^u \Big|_4^9$$

$$= \frac{1}{6} (e^9 - e^4)$$