

On the Normativity of Logic

Is logic normative for thought? In other words, does logic impose norms on how we ought (or ought not) to reason? John Macfarlane (2004) and Hartry Field (2009) answer affirmatively, both responding to Gilbert Harman's (2002) claim that logic and reasoning are distinct and do not have a normative connection. As Harman aptly acknowledges, it is not reasonable to demand that agents believe all the logical implications of their beliefs; to do so, the agent would have to be a logically omniscient super-genius. Furthermore, even if beliefs $A_1, \dots, A_n$ logically imply B , it is sometimes reasonable to reject B and $A_1, \dots, A_n$ in light of sufficient evidence for *not* B . The question is, then, whether there is a weaker sense in which logic is normative for reasoning. In this paper, I draw from Macfarlane (2004) and Field (2009) to argue that, despite Harman's objections, there is a defensible way of describing the normative connection between logic and reasoning.

Section One: What is Logic?

It would be foolish to ask whether logic is normative without first defining what logic is. Logic is the study of *logical consequence* (i.e. *validity*). Defining 'logic' is thus a matter of defining 'validity.' Definitions of validity fit roughly into two categories—*model-theoretic* definitions, and (as I have chosen to call them) *model-independent* definitions. Here, I elaborate on the two categories and briefly argue that what logicians do is better described by definitions of the model-independent variety.

Model-theoretic definitions of validity define the term in terms of how it is stipulatively defined within a formal system. Describing this view, Theodore Sider (2009) writes,

[...] one chooses a formal language, defines a notion of a model (or interpretation) for the chosen language, defines a notion of truth-in-a-model for sentence of the language, and then finally represents logical consequence [i.e. *validity*] for the chosen language as truth-preservation in models (pg. 10).

Sider goes on to proclaim that this definition is not a theory of *genuine logical validity*, in the sense that it says nothing about truth-preservation or logical consequence independently of any formal language. Indeed, this view defines validity in purely formal terms, where B is a logical consequence of $A_1, \dots, A_n$ if and only if B is true in any model in which $A_1, \dots, A_n$ are all true. Given a model-theoretic definition of validity, the claim that logic is normative is a non-starter. If logic is simply the study of arbitrarily stipulated definitions of validity, then it is unclear why it should have any normative role.

On the other hand, model independent definitions of validity do constitute theories of genuine logical validity, as they define the term independently of any formal system. On this

view, validity is not something that logicians arbitrarily stipulate, but rather, something logicians study and try to capture the nature of. One such definition might build from the model-theoretic approach, positing that validity in a specific system (e.g. classical logic) *is* genuine logical validity. Another type of model-independent definition defines validity in terms of the substitution of expressions for non-logical expressions. For instance, W. V. O. Quine argues,

B is a logical consequence of $A_1, \dots, A_n$ if and only if there is no way to (uniformly) substitute expressions for nonlogical expressions in B and $A_1, \dots, A_n$ so that $A_1, \dots, A_n$ all become true but B does not (Quine, 1960, as paraphrased by Sider, pg. 11).

Finally, validity could be cashed out in metaphysical terms, where logical consequence is defined in terms of modal terms such as possibility and necessity. Whatever the case may be, the key distinction is that the model-theoretic definitions of validity define the term formally, relative to a specific logical system, whereas model-independent definitions of validity define the term independently of any system or formal language. The goal of building formal systems is to capture the model-independent notion of validity in formal terms.

I hold that the model-independent definitions of validity better describe what logicians do when they study logic. Validity is *truth by virtue of form*, not content. For instance, “It is raining” is logically implied by “it is raining” in virtue of the fact that $P \Rightarrow P$ regardless of which declarative sentences are substituted for P. Contrastingly, “John is an unmarried man” is implied (but not logically) by “John is a bachelor” in virtue of the content of ‘bachelor.’ Hence, it seems that the goal of logic is to capture *truth-relevant* aspects of the *form* of natural-language sentences. By representing sentences as primitives, classical logic fails to account for the object-predicate structure of natural language sentences which is relevant to truth-preservation; hence the reason logicians invented predicate logic. Similarly, by representing modal terms as predicates, predicate logic fails to account for the structure of modal terms in natural language; hence the reason that logicians invented modal logic. The takeaway is that logicians do not arbitrarily create formal systems just to stipulate new definitions of validity. Rather, logicians create their systems to capture what makes arguments in natural language preserve truth in virtue of their form. Thus, logic is best defined using a model-independent definition of validity. This is important for present purposes, because unlike the model-theoretic definitions, model-independent definitions allow for the possibility that logic provides normative constraints on reasoning.

Section Two: Harman’s Critique

Given that logic is defined as the study of validity, where ‘validity’ is described by a model-independent definition (i.e. as *genuine logical validity*), what can be said about the normativity of logic? Harman (2002) argues that there is no normative connection; logic is the abstract study of truth-preservation by virtue of form. We may try to have the right views about logic in the same way that we try to have the right views about any of the sciences, but logic plays no special normative role. Motivating this view, Harman points to (at least) four key

problems with the supposed normative role of logic—change of view, relevance, excessive demands, and the preface paradox. Each of these problems highlights the fact that it is not always reasonable for agents to believe all the logical implications of their beliefs. Let us consider each of these problems, in turn.

Change of View

As I stated in the introduction, the fact that $A_1, \dots, A_n$ imply B does not itself imply that if you believe $A_1, \dots, A_n$, then you ought to believe B . In fact, there are some cases in which it is rational to reject B and change one's view about $A_1, \dots, A_n$ in light of evidence that it is not the case that B . For instance, suppose Elwood believes that "Jeff is either in his office or at home" and that "Jeff is not in his office." If Elwood then sees Jeff at the grocery store, it is rational for him to reject the claim that "Jeff is at home" and change his view about the claim that "Jeff is either in his office or at home." Furthermore, it is presumably irrational for Elwood to maintain his belief that "Jeff is at home," despite the fact that it follows from his prior beliefs (given the assumption that disjunctive syllogism tracks genuine logical validity).

Relevance

Suppose that logic is normative. If $A_1, \dots, A_n$ imply B , then if you believe $A_1, \dots, A_n$, you ought to believe B . If the definition of validity in classical logic, predicate logic, or any logic that allows for introduction rules, captures the definition of genuine logical validity, then if an agent possesses even a single belief, it follows that they are also obligated to believe an infinite number of seemingly irrelevant claims. For instance, assuming that or-introduction (i.e. A implies A or B) tracks genuine logical validity, then if Elwood believes that "Jeff is at the grocery store," Elwood should also believe that "Jeff is at the grocery store or dragons exist," that "Jeff is at the grocery store or *Black Myth: Wukong* is a great game," and an infinite number of other beliefs, none of which are necessarily relevant to Elwood. To spell out the reductio argument here, it would obviously be irrational for an agent to believe an infinite number of irrelevant beliefs. Hence logic is not normative in the sense described above.

Preface Paradox

Presumably, if logic is normative for reasoning, then agents should not hold contradictory beliefs. But there are cases where it is rational to hold contradictory beliefs. Consider the classic preface paradox. If Richard writes a well-researched book and believes each claim he makes in the book, it is nevertheless rational for him to admit that at least one claim in the book is likely false. This seems to be a logical contradiction and yet it is not irrational to allow it.

Excessive Demands

Holding that agents ought to believe all of the logical implications of their beliefs is an excessive demand to place on any non-ideal agent. As the relevance objection illustrates, such a constraint implies that agents ought to hold an infinite number of irrelevant beliefs. Additionally, the constraint requires that agents are obligated to recognize all of the unobvious implications of

their beliefs. Such a demand essentially amounts to the claim that agents should be logically omniscient super-geniuses (assuming *ought* implies *can*).

Section Three: Responding to Harman's Critique

All of the above problems provide good reason to reject the claim that if $A_1, \dots, A_n$ imply B, then if you believe $A_1, \dots, A_n$, you ought to believe B. Harman concludes that the above problems imply that logic has no normative connection to our reasoning. But, as Macfarlane (2004) points out, we need not go so far. There are other weaker ways of construing the normative connection between logic and reasoning which are not subject to the objections raised above, corresponding to different formulations of what Macfarlane calls *Bridge Principles*. Each bridge principle takes the abstract form of (BP).

(BP) If $A_1, \dots, A_n \Rightarrow B$, then $N(\alpha(A_1), \dots, \alpha(A_n), \beta(B))$

Roughly speaking, BP states that if $A_1, \dots, A_n$ logically implies B, then some normative claim N about the doxastic attitudes α and β one should hold towards $A_1, \dots, A_n$ and B follows. Here, to keep things simple, let's assume the doxastic attitude is *believing*. Ultimately, BP provides a schema for mapping out the space of possible bridge principles. The goal is to identify a bridge principle which avoids all of the objections raised above.

Specific elaborations of BP vary along three main axes—type of deontic operator, polarity, and scope of deontic operator. *Type of deontic operator* refers to the type of deontic demand that takes the place of N. Macfarlane considers three possibilities—(o)bligations, (p)ermissions, or (r)easons for belief. *Polarity* refers to whether the deontic demands are obligations/permissions/reasons to *believe* (+) or merely obligations/permissions/reasons not to *disbelieve* (-). Finally, *scope of deontic operator* refers to how the deontic operator is embedded in the consequent of (BP). Again, there are three possibilities—the deontic operator can range over just the (c)onsequent (i.e. if you believe A, then you ought to believe B), (b)oth the antecedent and the consequent (i.e. if you ought to believe A, then you ought to believe B), and finally, over the (w)hole antecedent (i.e. it ought to be the case that if you believe A then you believe B). In addition to these three axes of variation, each bridge principle can be weakened by changing the antecedent of BP from 'if $A_1, \dots, A_n \Rightarrow B$ ' to 'if you know that $A_1, \dots, A_n \Rightarrow B$ '. Note that the letters in parenthesis allow for a quick and easy method of notating the different bridge principles. For instance, Wp+ denotes the following bridge principle: "if $A_1, \dots, A_n \Rightarrow B$, then you have permission for it to be the case that if you believe $A_1, \dots, A_n$, then you believe B." Now, to weaken the antecedent, we can write (k). So, Bo-k, for instance, denotes, "if you know that $A_1, \dots, A_n \Rightarrow B$, then if you ought not disbelieve $A_1, \dots, A_n$, you ought not disbelieve B."

Following Field (2009), the change of view problem gives compelling reason to reject the bridge principles whose deontic operator applies only to the consequent (pg. 253). For instance, consider (Co+k).

(Co+k) If you know that $A_1, \dots, A_n \Rightarrow B$, then if you believe $A_1, \dots, A_n$ you ought to believe B.

Recall that bridge principles of this sort would require that we believe the logical implications of our beliefs even in light of strong evidence to the contrary. The obvious solution is to widen the scope of the deontic operator, yielding something like (Wo+k).

(Wo+k) If you know that $A_1, \dots, A_n \Rightarrow B$, then you ought not believe $A_1, \dots, A_n$ without believing B.

According to this principle I can rationally subtract from my prior beliefs and hence, change my view in light of the evidence.

Now, to tend to the relevance objection, let us again follow Field (pg. 257), and introduce a distinction between implicit and explicit belief. Recall that the relevance objection states that we should not clutter our mind with an infinite number of irrelevant beliefs. Wo+k is subject to this critique, because it states that you ought not accept a set of beliefs without also accepting their consequences. Hence, if you accept that “Jeff is at the grocery store”, then Wo+k implies that you should also accept that “Either Jeff is at the grocery store or dragons exist.” We can avoid this objection by altering Wo+k to produce Wo+k*.

(Wo+k*) If you know that $A_1, \dots, A_n \Rightarrow B$, then you ought not explicitly believe $A_1, \dots, A_n$ without at least implicitly believing B.

Explicit beliefs are beliefs that are directly stored in memory, whereas implicit beliefs are those that one merely has a disposition to believe if they consider them. Hence, if you believe that “Jeff is at the grocery store,” then you need only believe that “Either Jeff is at the grocery store or dragons exist” if you subject it to consideration.

The preface objection can be dealt with by altering $A_1, \dots, A_n$ in the antecedent to $A_1 \wedge \dots \wedge A_n$. Hence, Wo+k* can be altered to yield Wo+k**.

(Wo+k**) If you know that $A_1, \dots, A_n \Rightarrow B$, then you ought not explicitly believe $A_1 \wedge \dots \wedge A_n$ without at least implicitly believing B.

There is a difference between independently believing each of $A_1, \dots, A_n$ and believing the conjunction $A_1 \wedge \dots \wedge A_n$. The preface paradox dissolves if one believes the former and not the latter, because Richard’s belief that at least one of the claims in his book is false does not contradict belief in any specific claim of the book, but rather, with the belief in the conjunction of every claim in the book.

Finally, consider the excessive demands objection, which states that requiring people to believe all of the logical consequences of their beliefs is too strict of a demand; you would have to be a genius to recognize all of said logical consequences, even implicitly so. So far, all of the bridge principles we have considered are of the (k) variety, meaning that we are only normatively bound by the logical consequences we already know. This avoids the excessive demands objection, but it seems far too weak. Logicians study logical consequence not to impose new normative demands on themselves, but to learn about what they should believe regardless of what they know. To strengthen the bridge principle while avoiding the excessive demand objection, there are a couple of potential strategies. First, change the antecedent from “if you know that $A_1, \dots, A_n \Rightarrow B$ ” to “if $A_1, \dots, A_n$ obviously imply B ”, where ‘obviously’ is defined by some measure of *inferential closeness*. Second, you can complexify the bridge principle by introducing degrees of belief. Macfarlane adopts both approaches, ultimately advocating for something akin to the following:

If B follows from $A_1, \dots, A_n$ by a single application of a rule on the list, then one ought to impose the constraint that $P(B)$ is to be at least $P(A_1) + \dots + P(A_n) - (n-1)$, in any circumstance where $A_1, \dots, A_n$ and B are in question.

These considerations are beyond the scope of the current paper, but the latter should suffice to show that there are defensible ways of maintaining a bridge principle which is sufficiently strong, and yet which avoids Harman’s excessive demands objection.

Conclusion:

Harman points out several problems with the supposed normative connection between logic and reasoning. However, following Macfarlane (2004) and Field (2009), each of these problems can be avoided by positing a bridge principle which is neither too weak nor too strong. Hence, there is still a case to be made for the normative connection between logic and reasoning.

Though it’s not within the scope of this paper to discuss in detail, bridge principles also provide a way of settling disputes about logical validity. For instance, consider the dispute between classicalists, who claim that classical logic validity is genuine logical validity, and relevantists, who claim that introduction rules (e.g. $A \Rightarrow A$ or B) are at odds with genuine logical validity, and hence that our logical system should exclude them. Without using bridge principles, relevantists and classicalists are stuck with using brute appeals to intuition to settle their disputes. Using bridge principles, the debate can instead be settled by appealing to more substantial and widespread intuitions about what is rational. For instance, the relevantists could defend their position by defending a principle like $Cp+$, which states that logical implications merely give us permissions, rather than obligations, to believe what follows from our prior beliefs. Classicalists, on the other hand, could argue against relevantists by rejecting $Cp+$.

In sum, contrary to Harman (2002), there is good reason to think that there is a normative connection between logic and reasoning. Macfarlane (2004) provides a useful way of thinking about the problem, but the issue is far from settled.

Works Cited

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