Definition: Two linear systems (and their corresponding augmented matrices) are said to be **equivalent** if they have the same solution set. For example, these systems have the same solution set, $(x_1, x_2) = (1, 2)$.

$$\begin{cases} 3x_1 - 2x_2 = -1 \\ x_1 + 4x_2 = 9 \end{cases} \qquad \begin{cases} 3x_1 - 2x_2 = -1 \\ 2x_1 + x_2 = 4 \end{cases}$$

Therefore these augmented matrices are **equivalent**, which we denote with $\sim$.

$$\left[\begin{array}{cc|c} 3 & -2 & -1 \\ 1 & 4 & 9 \end{array}\right] \sim \left[\begin{array}{cc|c} 3 & -2 & -1 \\ 2 & 1 & 4 \end{array}\right]$$

Note: these augmented matrices are NOT equal. Two matrices are equal if and only if ALL their entries are equal.

$$\left[\begin{array}{cc|c} 3 & -2 & -1 \\ 1 & 4 & 9 \end{array}\right] \neq \left[\begin{array}{cc|c} 3 & -2 & -1 \\ 2 & 1 & 4 \end{array}\right]$$

Warm-Up Exercise: Consider whether each of the following matrix manipulations must keep the solution set the same or might result in a different solution set for the corresponding linear system.

1. **Swapping two rows.** For example,

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & 3 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 3 & 5 \\ 1 & 2 & 4 \end{array}\right]$$

$$\begin{cases} x + 2y = 4 \\ x + 3y = 5 \end{cases} \qquad \begin{cases} x + 3y = 5 \\ x + 2y = 4 \end{cases}$$

A. Solutions must be the same B. Solutions might be different

2. **Swapping two columns.** For example,

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & 3 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & 1 & 4 \\ 3 & 1 & 5 \end{array}\right]$$

$$\begin{cases} x + 2y = 4 \\ x + 3y = 5 \end{cases} \qquad \begin{cases} 2x + y = 4 \\ 3x + y = 5 \end{cases}$$

A. Solutions must be the same B. Solutions might be different

3. **Adding a constant to every term of a row.** For example,

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & 3 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1+3 & 2+3 & 4+3 \\ 1 & 3 & 5 \end{array}\right]$$

$$\begin{cases} x + 2y = 4 \\ x + 3y = 5 \end{cases} \qquad \begin{cases} 4x + 5y = 7 \\ x + 3y = 5 \end{cases}$$

A. Solutions must be the same B. Solutions might be different

4. **Multiplying a row by a nonzero constant.** For example,

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & 3 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 3\cdot 1 & 3\cdot 2 & 3\cdot 4 \\ 1 & 3 & 5 \end{array}\right]$$

$$\begin{cases} x+2y=4 \\ x+3y=5 \end{cases} \qquad \begin{cases} 3x+6y=12 \\ x+3y=5 \end{cases}$$

A. Solutions must be the same B. Solutions might be different

5. **Adding one row to another row.** For example,

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & 3 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1+1 & 3+2 & 5+4 \end{array}\right]$$

$$\begin{cases} x+2y=4 \\ x+3y=5 \end{cases} \qquad \begin{cases} \end{cases}$$

A. Solutions must be the same B. Solutions might be different

6. **Replacing a column with zeros.** For example,

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & 3 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 1 & 0 & 5 \end{array}\right]$$

$$\begin{cases} x+2y=4 \\ x+3y=5 \end{cases} \qquad \begin{cases} \end{cases}$$

A. Solutions must be the same B. Solutions might be different

7. **Replacing a row with zeros.** For example,

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 1 & 3 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x+2y=4 \\ x+3y=5 \end{cases} \qquad \begin{cases} \end{cases}$$

A. Solutions must be the same B. Solutions might be different