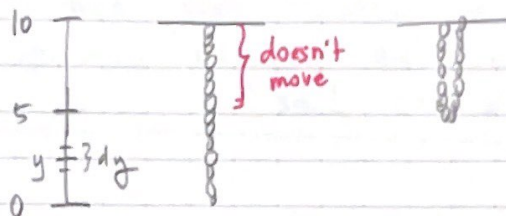


example: A 10ft chain weighs 25lbs and hangs from a ceiling. Find the work done to lift the hanging end to the ceiling so that it's level with the other end.



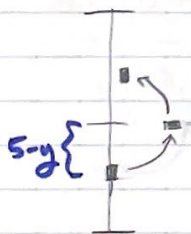
For $0 \leq y \leq 5$, consider a chunk of length dy .

Weight of chunk? Need the linear weight density.

$$\rho = \frac{\text{linear weight density}}{\text{total length}} = \frac{\text{total weight}}{\text{total length}} = \frac{25 \text{ lbs}}{10 \text{ ft}} = 2.5 \text{ lbs/ft}$$

$$\text{Then the chunk weighs: } \rho \cdot \text{length} = \boxed{2.5 dy \text{ lbs}}$$

$$dW = F \cdot d = (\text{weight}) (\text{dist. traveled}) = (2.5 dy) (?)$$



travels $5-y$ to be level
then another $5-y$ to where it stops

$$\boxed{d = 2(5-y)}$$

$$\Rightarrow dW = (2.5 dy) (2(5-y)) = 5(5-y) dy$$

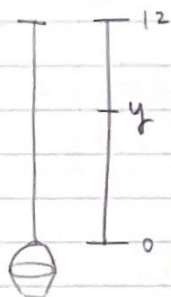
$$W = \int_0^5 5(5-y) dy = \left(25y - \frac{5}{2}y^2 \right) \Big|_0^5$$

$$= 125 - \frac{5}{2}(25) = \frac{125}{2} \text{ ft-lbs}$$

example: A leaky 10kg bucket of water is raised to a height of 12m at a constant speed with a rope that weighs 0.8 kg/m. Initially, the bucket contains 36kg of water but the water leaks at a constant rate and finishes draining just as the bucket reaches 12m. How much work is done?

First, since water leaks at a constant rate, how much water is left depends on the height of the bucket:

know at $y=0$, have 36 kg of water
at $y=12$, " 0 kg " "



loses 3 kg per meter

$$\Rightarrow \text{mass of water at height } y = 36 - 3y$$

At height y , the bucket weighs $\underbrace{((36-3y)+10) \cdot 9.8}_{\text{Force } F(y)}$

$$W_B = \int_0^{12} F(y) dy = \int_0^{12} 9.8(46-3y) dy$$

For the rope, $dW = F \cdot \text{dist} = m \cdot g \cdot \text{dist}$
 $= (0.8) dy \cdot (9.8)(12-y)$

$$W_R = \int_0^{12} 9.8(0.8)(12-y) dy$$

$$W = W_B + W_R \approx 3857 \text{ J}$$