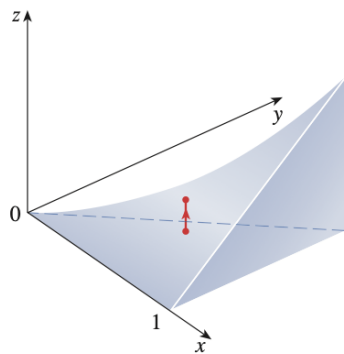


Rectangular Coordinates

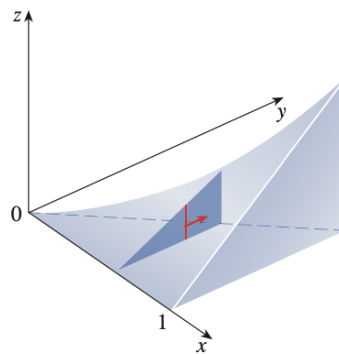
Example: E is the solid in the first octant bounded by $z = 12xy$, $y = x$ and $x = 1$.

$$\iiint_E f(x, y, z) \, dV = \int_0^1 \int_0^x \int_0^{xy} f(x, y, z) \, dz \, dy \, dx$$



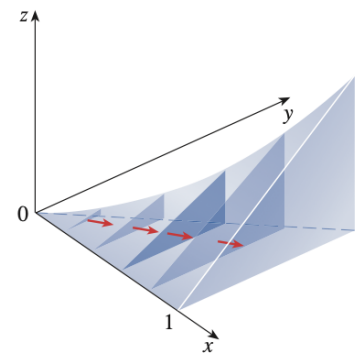
z varies from 0 to xy
while x and y are constant.

$$0 \leq z \leq xy$$



y varies from 0 to x
while x is constant.

$$0 \leq y \leq x$$



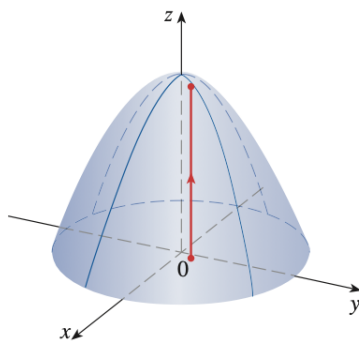
x varies from 0 to 1.

$$0 \leq x \leq 1$$

Cylindrical Coordinates

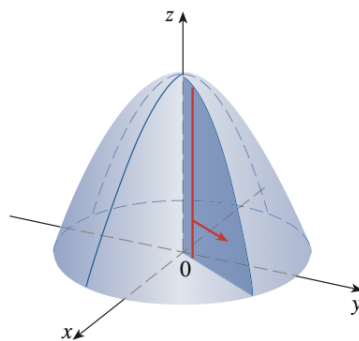
Example: E is the solid that lies under the paraboloid $z = 4 - x^2 - y^2$ and above the xy -plane.

$$\iiint_E f(x, y, z) \, dV = \int_0^{2\pi} \int_0^2 \int_0^{4-r^2} f(x, y, z) \, dz \, dr \, d\theta$$



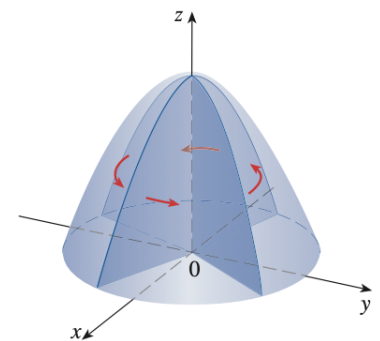
z varies from 0 to $4 - r^2$
while r and θ are constant.

$$0 \leq z \leq 4 - r^2$$



r varies from 0 to 2 while
 θ is constant.

$$0 \leq r \leq 2$$



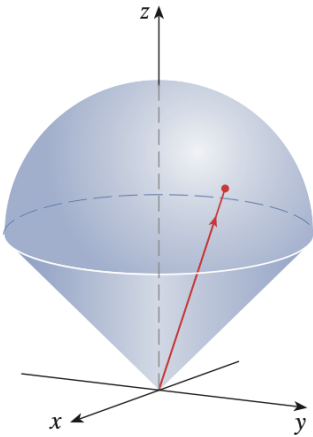
θ varies from 0 to 2π .

$$0 \leq \theta \leq 2\pi$$

Spherical Coordinates

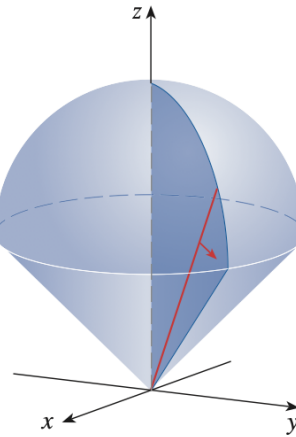
Example: E is the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.

$$\iiint_E f(x, y, z) \, dV = \int_0^{2\pi} \int_0^{\pi/4} \int_0^{\cos \phi} f(x, y, z) \, dz \, dr \, d\theta$$



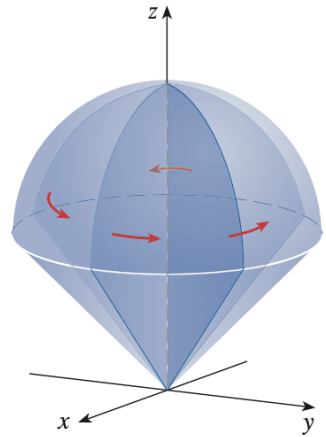
ρ varies from 0 to $\cos \phi$
while ϕ and θ are constant.

$$0 \leq \rho \leq \cos \phi$$



ϕ varies from 0 to $\pi/4$
while θ is constant.

$$0 \leq \phi \leq \frac{\pi}{4}$$



θ varies from 0 to 2π .

$$0 \leq \theta \leq 2\pi$$

