

I) Reading Secs. 1.4 focus on this
1.5
1.6

II) OBJECTIVES

A) Give an alternate, slightly more
intuitive - Derivation of Eq.
1.4.17

TEXT VERSION:

$$\Sigma(N, V, E) \approx \left(\frac{V}{h^3}\right)^N \frac{(2\pi m E)^{3N/2}}{(3N/2)!} \quad (1.4.17)$$

MY VERSION

$$(1.1) \quad \Sigma(N, V, E) \approx \left(\frac{V}{h^3}\right)^N \left(\frac{8mE}{N}\right)^{3N/2}$$

⊕ While these obviously differ,
associated
the ¹ MACROSCOPIC thermodynamics
is ~~same~~ the same!

B) solve Schrodinger's eqn for
a single particle in a 3-D
box; prove 1.4.5

lec 4 sps 15

(1.4.5)

$$\frac{h^2}{8mL^2} (n_x^2 + n_y^2 + n_z^2) = \epsilon$$

⊕ (1.4.5) is essential to
deriving $\Sigma(N, V, \epsilon)$

C) Derive ~~Fermi-Dirac~~ Equations

1.4.20 - 1.4.28

III DO OBS. B FIRST.

ASIDE QUICK OVERVIEW OF QUANTUM MECHANICS

- 1) THE DYNAMICAL STATE OF A SYSTEM IS ENCAPSULATED BY TWO ~~LOW~~ SYSTEM'S

WAVE FUNCTION, $\Psi = \Psi(x, t)$.
 $\oplus x$ ^{typically} corresponds to spatial coords and/or momenta

BJ

of the evolution

- 2) Ψ CAN BE THOUGHT OF AS A STATISTICAL GENERATING

FUNCTION = a mathematical entity that, in and of itself, has no apparent physical meaning (Kroemer, Quantum Mechanics); knowledge of

BJ

~~Ψ~~

Ψ allows one to, e.g., compute averages, $\langle A \rangle$, and variances, $\langle (A - \langle A \rangle)^2 \rangle = \langle A^2 \rangle - \langle A \rangle^2$

of a given experimental observables, A , where A can, e.g., correspond to a particle momentum, $\underline{p}$, particle position, $\underline{x}$, or particle angular momentum, $\underline{L}$. (More Below on $\langle A \rangle$, $\text{var}(A)$)
~~at given ψ~~

~~$\langle A \rangle = \int \psi^* A \psi$~~

3) The Time evolution of ψ for a given system is governed (we postulate) by Schrödinger's eqn:

$$\boxed{\hat{H}\psi = i\hbar\psi_t} \quad (26.1)$$

(20)

where H is the system

Hamiltonian:

$$(20.1) \quad H = \begin{array}{cc} \text{system} & + & \text{system} \\ \text{Kin.} & & \text{Pot.} \\ \text{Energy} & & \text{Energy} \end{array}$$

$$\hbar = \frac{h}{2\pi}$$

h = Boltzmann constant

- a) Two fundamental postulates of QM are the Planck - Einstein - de Broglie relations, which ~~assert~~ posit that ~~the~~ two particle-like properties, Energy and Momentum, E and p , are ~~the~~ linearly proportional to two

wave-like properties, wave
frequency, ω , and wave number,
 $\underline{k}$!

(2d.1) $\boxed{\begin{array}{l} E = \hbar \omega \\ \underline{p} = \hbar \underline{k} \end{array}} \quad (= \frac{h}{2\pi} 2\pi \nu = h\nu)$

(See Kroemer, sec. 1.1.2)

b) See Kroemer Ch. 1 for
an accessible derivation of
schrodinger's eqn;

i) associate w/ a particle
of mass m a wave
fn, ψ , of plane wave
form

⊕ we are trying to
marry the intuitive
ideas of a free-
free ptcl. moving
as a plane
wave.

⊕ Thus, guess that pte
can be represented as
a plane wave:

$$(a0) \quad \boxed{\psi(x,t) = Ae^{i(kx - \omega t)}}$$

iii) postulate that the PEB
relns apply to the pte.

$$(a1) \quad E = \hbar \omega$$

$$(a2) \quad p = \hbar k \quad (\text{here } p_x = \hbar k_x)$$

AND recognize that for
a pte moving
under no forces:

$$(a3) \quad E = \frac{mv^2}{2} = \frac{p^2}{2m}$$

Thus, combining (a1)-(a3) we
get a dispersion rela for

the matter wave $\therefore$ momentum associated w/ Fourier comp
 $\downarrow$ single comp $\downarrow$ single comp
 $\downarrow$ energy of single Fourier component $\downarrow$ single comp

$$E = \hbar \omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$$\therefore \boxed{\omega(k_x) = \frac{\hbar k_x^2}{2m}} \quad (a4)$$

Lecl. 4 Sep 13
(29)

iv) Next, note from (40);

and

$$\begin{cases} \hbar i \psi_t = \hbar A i (-i\omega) e^{i(x)} = A \hbar \omega e^{i(x)} \\ -\hbar^2 \psi_{xx} = \oplus \hbar^2 A k_x^2 e^{i(x)} \end{cases} \quad (45)$$

$\therefore$ combining (45) w/ (44):

$$\left(\frac{\hbar^2}{2m} \right) A A k_x^2 e^{i(x)} = A \hbar \omega e^{i(x)}$$

$\oplus$ $\boxed{\hbar \omega = \frac{\hbar^2 k_x^2}{2m}}$ Dispersion
reln for
force-free
ptcl. moving
in x-dir

or $\Rightarrow \underbrace{\hbar \omega}_{\psi} A e^{i(x)} = \frac{\hbar^2 k_x^2}{2m} \underbrace{A e^{i(x)}}_{\psi}$

or $\Rightarrow -i \hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi$

or (e.1) $\boxed{i \hbar \psi_t = \frac{\hbar^2}{2m} \psi_{xx}}$

V. 1
Schrodinger
eqn

$\oplus$ (e.1) linear

$\oplus$ ANY soln of (e.1) satisfies
the P.E.D.B. relns.

$\oplus$ since (e.1) linear, ~~any~~ superposition

(2A) Lec 4 Sp 915

Can be used to construct solns
to more complicated problems.

C) Connection between a System
Dispersion
Relation, Hamiltonian
Operator, DIFFERENTIAL OPERATORS
Form of Schrodinger's
Eqn

GENERAL

C1) A S.E. can be derived
From a system's
CM P.E.d.B.-based
Dispersion relation.

i) For the case where
a ptcl. is subjected
to a conservative
force, $F = -\nabla V$.

$$H = \frac{1}{2} m \underline{v} \cdot \underline{v} + V(\underline{r}, t) \\ = \frac{1}{2m} \underline{p} \cdot \underline{p} + V(\underline{r}, t)$$

2) From the PEBB relns

$$E \in H = \hbar \omega$$

$$\underline{P} = \hbar \underline{K}$$

we have:

(PEBB postulate)

$$\text{or } \left[\frac{\hbar^2}{2m} \underline{K} \cdot \underline{K} + V(\underline{r}, t) = \hbar \omega \right] \quad (9.1)$$

⊕ system dispersion reln.

3) multiply (9.1) by ψ

where ψ is a single Fourier mode (= plane wave)

$$\left[\psi = A e^{i(\underline{K} \cdot \underline{r} - \omega t)} \right] \quad (9.2)$$

$$\Rightarrow \left[\left(\frac{\hbar^2}{2m} \underline{K} \cdot \underline{K} + V \right) \psi = \hbar \omega \psi \right] \quad (9.3)$$

i.e. By inspection, we recognize that (9.3) implies / ~~leads~~ leads to the following operator definitions:

~~Handwritten scribbles~~

$$i) \left[\frac{\hbar^2}{2m} \nabla^2 = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \right]$$

$$ii) \left[\frac{\hbar^2}{2m} \nabla^2 \right]$$

$$iii) \left[\hat{H} = \frac{\hat{P} \cdot \hat{P}}{2m} + \hat{V} \right]$$

iv) From P.E.D.B. relation $P = \hbar K$

$$\left[\hat{P} = \hbar \hat{K} = -i\hbar \nabla \right]$$

$$\text{Also } \left[\hat{P}^2 = \hat{P} \hat{P} = -\hbar^2 \nabla^2 \right]$$

∴ (9.3) can apparently be

expressed in operator form;

$$\left[\left(\frac{\hat{P}^2}{2m} + \hat{V} \right) \psi = i\hbar \frac{\partial \psi}{\partial t} \right] \quad (9.4)$$

$$\text{or } \left[\hat{H} \psi = i\hbar \psi_t \right] \quad (9.5)$$

Verify that (9.4) ~~reproduces~~
reproduces the dispersion relation
in (9.3):

$$\frac{\hbar^2}{2m} \nabla^2 A e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$+ V(x, t) A e^{i(\dots)} = i \hbar \frac{\partial}{\partial t} A e^{i(\dots)}$$

$$\Rightarrow \frac{\hbar^2}{2m} (k_x^2 + k_y^2 + k_z^2) A e^{i(\dots)} + V A e^{i(\dots)} = \hbar \omega A e^{i(\dots)}$$

$$\boxed{\frac{\hbar^2 k^2}{2m} + V(x, t) = \hbar \omega} \quad \checkmark$$