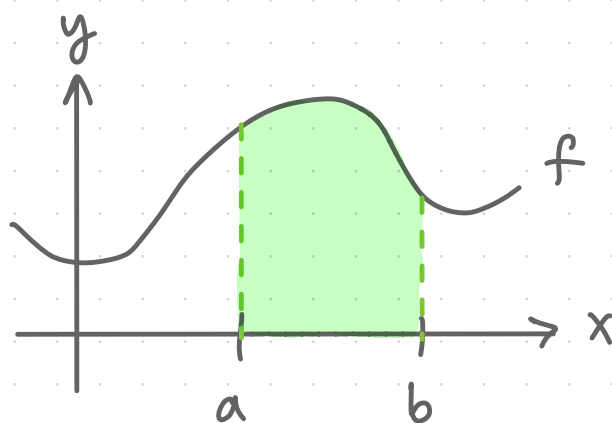
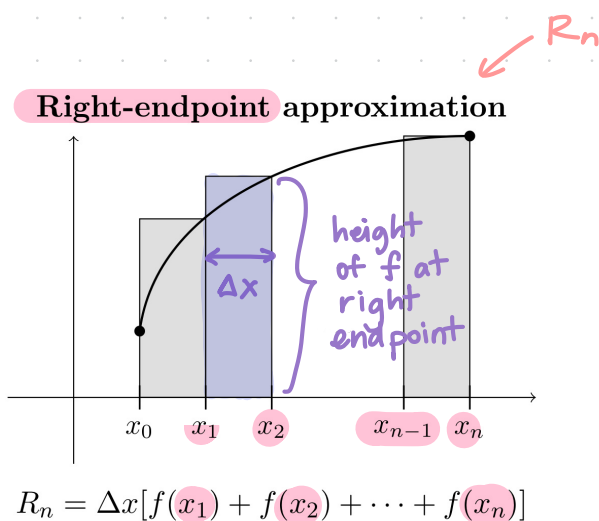
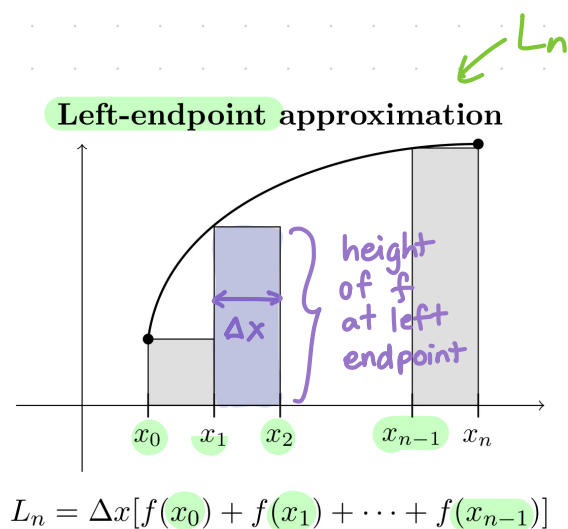


Approximating Definite Integrals

Recall: $\int_a^b f(x) dx$ is the area under the curve (graph of f) on the interval $[a, b]$

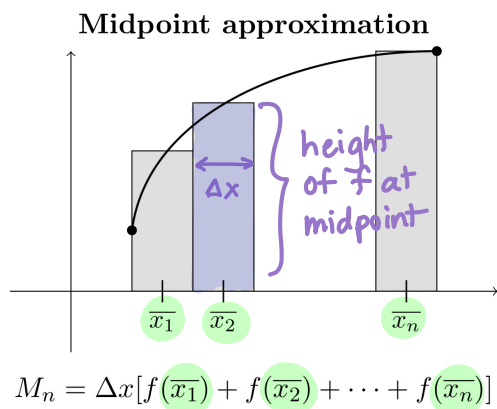


When we define $\int_a^b f(x) dx$ in Calc 1, we usually start with the left or right endpoint approximations (AKA the left/right Riemann sums)



Here n is the number of rectangles used in the approximation
 Δx is the width of the rectangles (Note: $\Delta x = \frac{b-a}{n}$)

We could also use the midpoint approximation (or Midpoint Rule, M_n)

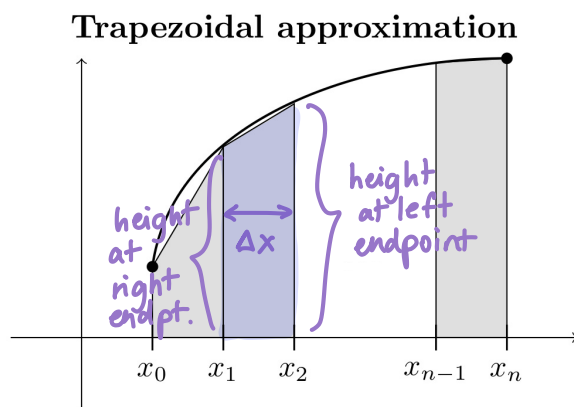
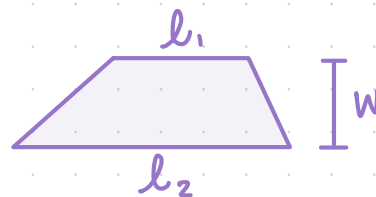


We use the notation $\bar{x}_i$ for the i th midpoint:

$$\bar{x}_i = \frac{x_{i-1} + x_i}{2} \quad (\text{average of the endpoints})$$

The Trapezoidal Rule uses trapezoids instead of rectangles.

$$\text{Area} = \underbrace{\left(\frac{l_1 + l_2}{2} \right)}_{\text{average of lengths}} \cdot \underbrace{w}_{\text{width}}$$



$$T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + f(x_n)]$$

uses all the endpoints

Why this formula?

$$\begin{aligned} T_n &= \left(\frac{f(x_0) + f(x_1)}{2} \right) \Delta x + \left(\frac{f(x_1) + f(x_2)}{2} \right) \Delta x + \left(\frac{f(x_2) + f(x_3)}{2} \right) \Delta x + \dots + \left(\frac{f(x_{n-1}) + f(x_n)}{2} \right) \Delta x \\ &= \frac{\Delta x}{2} \left(\underbrace{f(x_0)}_{\text{just one}} + \underbrace{f(x_1) + f(x_1)}_{2f(x_1)} + \underbrace{f(x_2) + f(x_2)}_{2f(x_2)} + \dots + \underbrace{f(x_{n-1}) + f(x_n)}_{\text{just one}} \right) \\ &= \frac{\Delta x}{2} (f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)) \end{aligned}$$

Sum of areas of the trapezoids