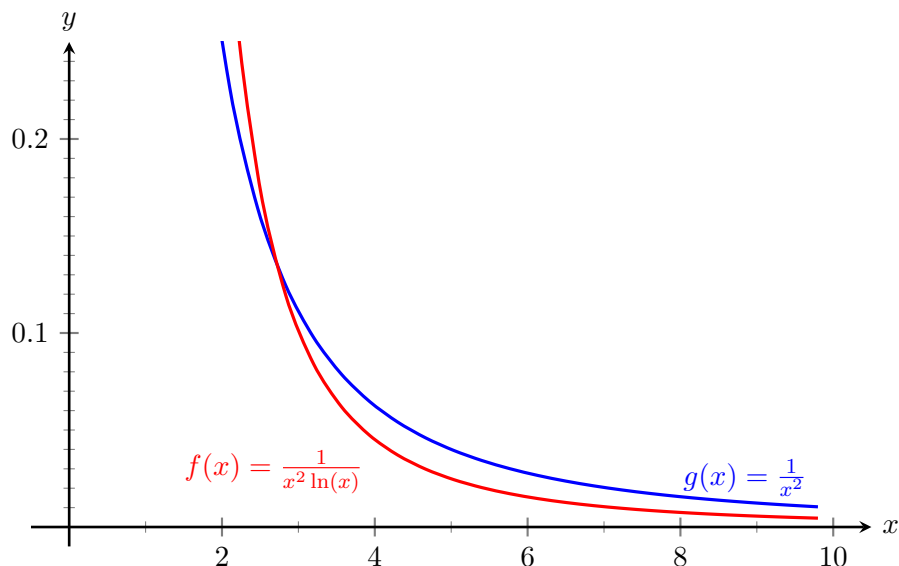


1. Use technology to help you draw the graphs of $f(x) = \frac{1}{x^2 \ln(x)}$ and $g(x) = \frac{1}{x^2}$. Pay attention to any intersection points, and pay attention to which function has higher values.

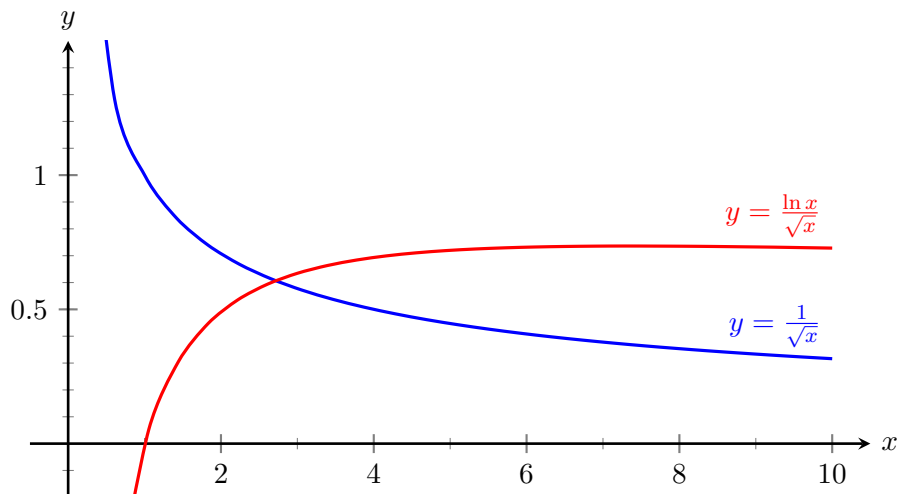


2. Using the graph you just made:

(a) Do you think $\int_3^{\infty} \frac{1}{x^2 \ln(x)} dx \leq \int_3^{\infty} \frac{1}{x^2} dx$ is true? Why or why not?

- (b) What does this suggest about the convergence/divergence of the two integrals and why?

3. Use technology to help you draw the graphs of $f(x) = \frac{\ln(x)}{\sqrt{x}}$ and $g(x) = \frac{1}{\sqrt{x}}$. Do these functions both have horizontal asymptotes?



4. Using the graph you just made:

(a) Do you think $\int_3^\infty \frac{1}{\sqrt{x}} dx \leq \int_3^\infty \frac{\ln(x)}{\sqrt{x}} dx$ is true? Why or why not?

- (b) What does this suggest about the convergence/divergence of the two integrals and why?

The punchline:**Comparison Theorem for Integrals**

If f and g are continuous functions with $f(x) \geq g(x) \geq 0$ for $x \geq a$, then

(a) If _____ is convergent, then _____ is convergent.

(b) If _____ is divergent, then _____ is divergent.

If we know $\int_a^\infty f(x) dx$ diverges, what can we conclude about $\int_a^\infty g(x) dx$?

If we know $\int_a^\infty g(x) dx$ converges, what can we conclude about $\int_a^\infty f(x) dx$?

Putting it into practice:

5. Consider the integral $\int_2^\infty \frac{\cos^2(x)}{x^2} dx$.

(a) A good comparison function is:

(b) What is the relationship between your comparison function and $\frac{\cos^2(x)}{x^2}$?

(c) What conclusion can you make using the Comparison Theorem?

6. Consider the integral $\int_1^{\infty} \frac{1}{\sqrt{x^2 + 1}} dx$.

(a) A good comparison function is:

(b) What is the relationship between your comparison function and $\frac{1}{\sqrt{x^2 + 1}}$?

(c) Will this inequality help you determine if $\int_1^{\infty} \frac{1}{\sqrt{x^2 + 1}} dx$ is convergent or divergent?