Adaptive Algorithm for Linear Systems with Input Delay[1]

Igor B. Furtat*,, Mikhail S. Tarasov**, Artem N. Nekhoroshikh****

**Institute for Problems of Mechanical Engineering Russian Academy of Sciences, 61 Bolshoy ave V.O., St.-Petersburg, 199178, Russia (Tel: +7-812-321-47-66; e-mail: cainenash@mail.ru).*
***ITMO University, 49 Kronverkskiy ave, Saint Petersburg, 197101, Russia.*

Abstract: The paper deals with the adaptive algorithm for single input single output (SISO) linear plants with any relative degree and input delay. The control system design is based on modified high-order tuners. The resulting algorithm ensures required accuracy of difference between the plant output and the reference signal. The modeling results illustrate the effectiveness of the proposed algorithm.

Keywords: Adaptive control, input delay, Lyapunov-Krasovskii functional, high-order tuner.

1. INTRODUCTION

The control of plants under uncertainties is one of a fundamental problem of automatic control. This problem becomes much more complicated if the plant has delay in the input signal. Moreover, the closed loop system can become unstable if delay is ignored (Kolmanovski and Nosov, 2009, Fridman, 2014).

First, the prediction problem was solved in (Smith, 1959). The solution of (Smith, 1959) is based on introduction of a special loop (Smith predictor) parallel to the plant. Another result is a Resvik controller (Kolmanovski and Nosov, 2009). However, if the relative degree of the plant model more than one, than Resvik controller can be not implementable. For discrete systems Tsypkin predictor is proposed in (Tsypkin, 1986). Note, that the solutions (Smith, 1959, Tsypkin, 1986) are obtained for stable plants with known parameters.

Other widely spread result is the method of (Manitius and Olbort, 1979). This algorithm allows to control of unstable plants with input delay. In (Kristic, 2009) application of the control algorithm (Manitius and Olbort, 1979) is considered with using backstepping method. The algorithms (Manitius and Olbort, 1979, Manitius and Olbort, 1979) are designed for plant with known parameters.

The paper (Lozano et. al, 2004) describes a resetting Smith predictor for control of linear plants under parametric uncertainties. In (Tsykunov, 2000) the adaptive modification of Smith predictor with one adjustable parameter is proposed for plants under measurable state vectors. In (Niculescu and Annaswamy, 2003) the results of (Smith, 1959, Manitius and Olbort, 1979) are generalized for control of single input single output (SISO) plants under parametric uncertainties with relative degree is not exceed two. The results of (Niculescu and Annaswamy, 2003) are generalized for control of stable plants with any relative degree in (Furtat, 2014). The papers (Bazylev, Pyrkin, 2013, Bazylev, Zimenko, Margun et al., 2014, Bazylev, Margun, Zimenko, 2014, Pyrkin, Bobtsov, Kolyubin et al., 2013) application of adaptive control under delays and constraints.

Unlike the described results, we will consider the problem of control of linear SISO plants with any relative degree and with input delay under parametric uncertainties. The control system design is based on modified high-order tuners, first proposed in (Tsykunov, 2006) and generalized for stable plants with input delay in (Furtat, 2014). Unlike higher order tuners proposed in (Morse, 1992, Nikiforov, 1999), the modified high order tuners have one adjustable parameter and allow reducing the dynamical order of the closed loop system.

The paper is organized as follows. The problem statement is presented in Section 2. In Section 3 the parameterization of the plant equation is considered. In Section 4 we design the control algorithm for SISO plants with any relative degree and input delay under parametric uncertainties. The proposed algorithm ensures small difference between plant output and reference model output. In Section 5 we consider simulation results. Concluding remarks are given in Section 6. Appendix A gives the proof of the control system.

2. PROBLEM STATEMENT

Consider a plant model in the form

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$$Q(p)y(t) = kR(p)u(t-h),$$
$$p^i y(0) = y_{i0}, \quad i = 0,...,n-1, \quad u(s) = 0, \quad s \in [-h, 0), \tag{1}$$

where $y(t) \in R$ is an output, $u(t) \in R$ is an input, $Q(p)$ and $R(p)$ are linear differential operators with unknown coefficients, $\deg Q(p) = n$, $\deg R(p) = m$, $k > 0$, y_{i0}, $i = 0, ..., n-1$ are unknown initial conditions, $p = d / dt$.

Assume that the coefficients of operators $Q(p)$, $R(p)$ and coefficient k belong to a known compact set Ξ. The polynomial $R(\lambda)$ is Hurwitz, where λ is a complex variable.

Consider the reference model

$$Q_m(p)y_m(t) = k_m R_m(p)r(t), \tag{2}$$

where $y_m(t) \in R$, $r(t)$ is a piecewise continious function, $k_m > 0$ is a known, $Q_m(p)$, $R_m(p)$ are known, $\deg Q_m(p) = n$, $\deg R_m(p) = m$, $Q_m(\lambda)$ and $R_m(\lambda)$ are Hurwitz polynomials.

The problem is to design the control system such that the following condition holds

$$|y(t) - y_m(t-h)| < \delta \text{ for } t > T, \tag{3}$$

where $\delta > 0$ is prespecified sufficiently required accuracy, $T > 0$ is a transient time.

3. PARAMETRIZATION OF A PLANT MODEL

Rewrite the operators $Q(p)$ and $R(p)$ as follows

$$Q(p) = Q_m(p) + \Delta Q(p), \quad R(p) = R_m(p) + \Delta R(p), \tag{4}$$

where $\Delta Q(p)$ and $\Delta R(p)$ are operators with unknown coefficients and the orders of these operators does not exceed $n-1$ and $m-1$ respectively.

Substituting (4) into (1) and taking into account (2), rewrite the equation for the tracking error $e(t) = y(t) - y_m(t-h)$:

$$e(t) = \frac{kR_m(p)}{Q_m(p)}\left[u(t-h) + \frac{\Delta R(p)}{R_m(p)}u(t-h) - \right.$$
$$\left. - \frac{\Delta Q(p)}{kR_m(p)}y(t) - \frac{k_m}{k}r(t-h)\right]. \tag{5}$$

Introduce the control law

$$u(t) = \frac{Q_m(p)}{R_m(p)}\bar{v}(t), \tag{6}$$

where $\bar{v}(t)$ is an estimate of the auxiliary control law $v(t)$. Taking into account control law (6), rewrite the equation (5):

$$e(t) = k\left[v(t-h) + \frac{\Delta R(p)}{R_m(p)}v(t-h) - \frac{\Delta Q(p)}{kQ_m(p)}y(t)\right.$$
$$\left. - \frac{k_m R_m(p)}{kQ_m(p)}r(t-h) + \varepsilon(t-h)\right], \tag{7}$$

where $R_m(p)\varepsilon(t) = R(p)\left(\bar{v}(t) - v(t)\right)$.

Introduce the filters

$$\dot{\theta}_1(t) = F_1\theta_1(t) + bv(t-h), \quad \theta_1(0) = 0,$$
$$\dot{\theta}_2(t) = F_2\theta_2(t) + by(t), \quad \theta_2(0) = 0, \tag{8}$$
$$\dot{\theta}_3(t) = F_2\theta_3(t) + br(t-h), \quad \theta_3(0) = 0.$$

Here $\theta_1(t) \in R^m$, $\theta_2(t) \in R^n$, $\theta_3(t) \in R^n$, F_1 and F_2 are matrices in Frobenius form with characteristic polynomials $R_m(\lambda)$ и $Q_m(\lambda)$ respectively, $b = [0, \ldots, 0, 1]^{\mathrm{T}}$.

Taking into account (8), rewrite equation (7) as follows

$$e(t) = k\left[v(t-h) + c_{01}^{\mathrm{T}}\theta_1(t) - c_{02}^{\mathrm{T}}\theta_2(t)\right.$$
$$\left. - c_{03}^{\mathrm{T}}\theta_3(t) + \varepsilon(t-h)\right], \tag{9}$$

where c_{01}, c_{02}, c_{03} are vectors of unknown constant parameters depending on the coefficients of $\Delta R(p)$, $\Delta Q(p)/k$ and $k_m R_m(p)/k$ accordingly.

Taking into account the first expression of (4), rewrite the plant (1) in the form

$$y(t) = \frac{kR(p)}{R_m(p)}v(t-h) - \frac{\Delta Q(p)}{Q_m(p)}y(t) + k\varepsilon(t-h)$$
$$= \bar{c}_{01}^{\mathrm{T}}\theta_1(t) - kc_{02}^{\mathrm{T}}\theta_2(t) + k\varepsilon(t-h), \tag{10}$$

where $\bar{c}_{01}$ is a vector composed of the coefficients of $kR(p)$. Substituting (10) into (8), we get

$$\dot{\theta}_2(t) = \left(F_2 - kbc_{02}^{\mathrm{T}}\right)\theta_2(t) + b\bar{c}_{01}^{\mathrm{T}}\theta_1(t) + kb\varepsilon(t-h). \tag{11}$$

Introduce the notation

$$c_0 = -\left[c_{01}^{\mathrm{T}} \; c_{02}^{\mathrm{T}} \; c_{03}^{\mathrm{T}}\right]^{\mathrm{T}}, \; w(t) = \left[\theta_1^{\mathrm{T}}(t) \; \theta_2^{\mathrm{T}}(t) \; \theta_3^{\mathrm{T}}(t)\right]^{\mathrm{T}}. \tag{12}$$

Taking into account (11) and (12), rewrite equations (8) and (9) in the form

$$\dot{w}(t) = Aw(t) + Bf(t-h) + D\varepsilon(t-h), \tag{13}$$
$$e(t) = k\left[v(t-h) - c_0^{\mathrm{T}}w(t) + \varepsilon(t-h)\right], \tag{14}$$

where

$$A = \begin{bmatrix} F_1 & 0 & 0 \\ b\bar{c}_{01}^{\mathrm{T}} & F_2 - kbc_{02}^{\mathrm{T}} & 0 \\ 0 & 0 & F_2 \end{bmatrix}, \; B = \begin{bmatrix} b & 0 \\ 0 & 0 \\ b & 0 \end{bmatrix}, \; D = \begin{bmatrix} 0 \\ kb \\ 0 \end{bmatrix},$$
$$f(t-h) = \left[v(t-h) \quad r(t-h)\right]^{\mathrm{T}}.$$

Let us find the solution of (13) in the form

$$w(t+h) = e^{Ah}w(t) + \int_{-h}^{0} e^{-Ag}Bf(t+g)dg$$
$$+ \int_{-h}^{0} e^{-Ag}D\varepsilon(t+g)dg. \tag{15}$$

Substituting (15) into (14), we get

$$e(t) = k\left[v(t-h) - \int_{-h}^{0}\beta_0^{\mathrm{T}}(g)f(t+g-h)dg\right.$$
$$\left. - \alpha_0^{\mathrm{T}}w(t-h)\right] + \varphi(t). \tag{16}$$

Here $\varphi(t) = \int_{-h}^{0}\gamma_0^{\mathrm{T}}(g)\varepsilon(t+g-h)dg + k\varepsilon(t-h)$,

$\alpha_0^{\mathrm{T}}(g) = c_0^{\mathrm{T}}e^{Ah}$, $\beta_0^{\mathrm{T}}(g) = c_0^{\mathrm{T}}e^{-Ag}B$, $\gamma_0^{\mathrm{T}}(g) = kc_0^{\mathrm{T}}e^{-Ag}D$ are new vectors and matrix of unknown constant parameters.

4. THE MODIFIED HIGH ORDER TUNERS FOR PLANT WITH INPUT DELAY

Introduce the auxiliary control law in the form

$$v(t) = \alpha^{\mathrm{T}}(t)w(t) + \int_{-h}^{0} \beta^{\mathrm{T}}(t, g) f(t+g)dg\,, \qquad (17)$$

where $\alpha(t)$ and $\beta(t, g)$ are adjustable vectors. Substitute (17) into (16) and rewrite (16) in the form

$$e(t) = k\Big[\big(\alpha(t-h) - \alpha_0\big)^{\mathrm{T}} w(t-h) + \int_{-h}^{0} \big(\beta(t-h, g) - \beta_0(g)\big)^{\mathrm{T}} f(t+g-h)dg\Big] + \varphi(t). \qquad (18)$$

For implementation of control law (6) consider the observer of (Atassi and Khalil, 1999). The equations of the observer are presented as follows

$$\dot{\xi}(t) = G_0\xi(t) + D_0(\bar{v}(t) - v(t))\,,\ \bar{v}(t) = L\xi(t)\,, \qquad (19)$$

where $\xi(t) \in R^{\gamma}$, $G_0 = \begin{bmatrix} 0 & I_{\gamma-1} \\ 0 & 0 \end{bmatrix}$, $I_{\gamma-1}$ is an identity matrix of order $\gamma - 1$, $D_0 = -\left[d_1\mu^{-1}, d_2\mu^{-2}, \ldots, d_{\gamma}\mu^{-\gamma}\right]^{\mathrm{T}}$, the coefficients $d_1, d_2, \ldots, d_{\gamma}$ are chosen such than the matrix $G = G_0 - DL$ is Hurwitz, $D = \left[d_1, d_2, \ldots, d_{\gamma}\right]^{\mathrm{T}}$, $L = [1, 0, \ldots, 0]$, $\mu > 0$.

Consider the vector $\overline{\eta} = \Gamma^{-1}(\xi - \sigma)$, where $\Gamma = diag\left\{\mu^{\gamma-1}, \mu^{\gamma-2}, \ldots, \mu, 1\right\}$, $\sigma = \left[v, \dot{v}, \ldots, v^{(\gamma-1)}\right]^{\mathrm{T}}$. Taking into account (19), take the derivative of $\overline{\eta}$

$$\dot{\overline{\eta}}(t) = \mu^{-1}G\overline{\eta}(t) + bv^{(\gamma)}(t)\,,\ \overline{\Delta}(t) = \mu^{\gamma-1}L\overline{\eta}(t)\,. \qquad (20)$$

Rewrite (20) in the form

$$\dot{\eta}(t) = \mu^{-1}G\eta(t) + q\dot{v}(t)\,,\ \overline{\Delta}(t) = \mu^{\gamma-1}L\eta(t)\,. \qquad (21)$$

Here $\eta_1 = \overline{\eta}_1$, $q = [\mu^{1-\gamma}, 0, \ldots, 0]^{\mathrm{T}}$.

Taking into account (21), rewrite equation (18) as follows

$$e(t) = k\Big[\big(\alpha(t-h) - \alpha_0\big)^{\mathrm{T}} w(t-h) - \int_{-h}^{0} \big(\beta(t-h, g) - \beta_0(g)\big)^{\mathrm{T}} f(t+g-h)dg\Big] + \varphi(t). \qquad (22)$$

Theorem. There exist $\mu_0 > 0$ such that for $\mu \le \mu_0$ the control system consisting of filters (8), control laws (6) and (17), observer (19) and adaptation algorithms

$$\dot{\alpha}(t) = -\rho_1 e(t)w(t-h),\quad \frac{\partial\beta(t, g)}{\partial t} = -\rho_2 e(t) f(t+g-h), \qquad (23)$$

ensures goal (3) and boundedness of all signals in the closed loop systems.

The proof of Theorem is in Appendix.

Remark. We note, that exact implementation of the integral term of (17) is impossible. However, we can use any numerical scheme for approximation of the integral term. Therefore, rewrite equations (17) and the second equation of (23) in the form

$$v(t) = \alpha^{\mathrm{T}}(t)w(t) + \sum_{i=0}^{q} m_i \beta^{\mathrm{T}}(t, iq^{-1}) f(t + iq^{-1} - h), \qquad (24)$$

$$\frac{\partial\beta(t, iq^{-1})}{\partial t} = -\rho_2 e(t) f(t + iq^{-1} - h),\ i = 0, 1, \ldots, q,$$

where the values m_i depend on the chosen numerical scheme for realization of the integral terms in (17), the parameter q determines the approximation precision.

5. EXAMPLE

Consider a plant model in the form

$$\left(p^3 + a_1 p^2 + a_2 p + a_3\right)y(t) = ku(t-h)\,. \qquad (25)$$

The set Ξ is determined by the following inequalities:

$$-3 \le a_1 \le -1, -3 \le a_2 \le -1, -3 \le a_3 \le 0.6, 0.5 \le k \le 1.5.$$

Let us consider the reference model

$$(p+1)^3 y_m(t) = 0.1 + 0.3\sin 0.2t,\ y_m(0) = \dot{y}_m(0) = \ddot{y}_m(0) = 0.$$

Since $R_m(p) = 1$, $Q_m(p) = (p+1)^3$, then the first filter of (8) is absent. Other filters are represented as follows

$$\dot{\theta}_2(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}\theta_2(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} y(t),\quad \theta_2(0) = 0,$$

$$\dot{\theta}_3(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}\theta_3(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t-h),\ \theta_3(0) = 0.$$

Therefore, $w(t) = \left[\theta_2^{\mathrm{T}}(t)\ \theta_3^{\mathrm{T}}(t)\right]^{\mathrm{T}}$.

Choose $D = [3\ 3\ 1]^{\mathrm{T}}$ and $\mu = 0.01$. Therefore, rewrite observer (19) in the form

$$\dot{\xi}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\xi(t) - \begin{bmatrix} 3\cdot 0.01 \\ 3\cdot 0.01^2 \\ 1\cdot 0.01^3 \end{bmatrix}\left(\bar{v}(t) - v(t)\right),$$

$$\bar{v}(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\xi(t),$$

where $\xi(t) = [\xi_1(t), \xi_2(t), \xi_3(t)]^{\mathrm{T}}$, $\xi(0) = 0$.

Let $\rho_1 = 2$ и $\rho_2 = 1.5$ in (23). As a result, adaptation algorithms (23) and control laws (6), (17) are presented as

$$\dot{\alpha}(t) = -2e(t)w(t-h),\ \alpha(0) = 0,$$

$$\frac{\partial\beta(t, g)}{\partial t} = -1.5e(t) f(t+g-h),\ \beta(0, g) = 0,$$

$$v(t) = \alpha^{\mathrm{T}}(t)w(t) + \int_{-h}^{0} \beta^{\mathrm{T}}(t, g) f(t+g-h)dg, \qquad (26)$$

$$u(t) = (p+1)^3 \bar{v}(t) = \xi_1(t) + 3\xi_2(t) + 3\xi_3(t) + \dot{\xi}_3(t).$$

Let us use (24) for implementation of the integral term of (26). Use the trapezoidal rule for approximation of the integral term of (26) for $q = 5$. Then, rewrite the second and the third equations of (26) as follows

$$v(t) = \alpha^{\mathrm{T}}(t)w(t) + 0.2h\Big[0.5\beta^{\mathrm{T}}(t,-h)f(t-2h) + 0.5\beta^{\mathrm{T}}(t,0)f(t-h) + \sum_{i=1}^{4}\beta^{\mathrm{T}}(t, iq^{-1})f(t+iq^{-1}-h)\Big],$$

$$\frac{\partial\beta(t, jq^{-1})}{\partial t} = -1.5e(t)f(t+jq^{-1}-h), \quad j = 0, 1, ..., 5.$$

Fig. 1 shows the simulation results of the plant output $y(t)$, the output of the reference model $y_m(t)$ for the following parameters of plant (25):

$$y(0) = \dot{y}(0) = \ddot{y}(0) = 1,\ k = 1,$$

$$a_1 = -1,\ a_2 = -2,\ a_3 = 0.6,\ h = 1 \text{ s}.$$

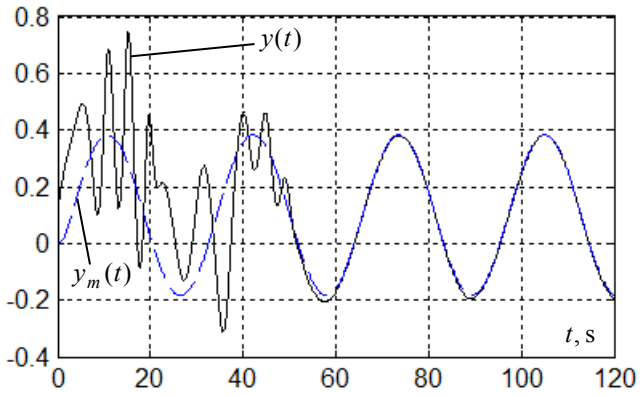


Fig. 1. The transients on the signals $y(t)$ (black solid curve) and $y_m(t)$ (blue dashed curve).

It follows from Fig. 1 that parametric uncertainties are compensated by the control system. The required accuracy $\delta = 0.01$ achieved after 70 s in Fig. 1 under constraints. We note that proposed algorithm can be successfully used for multi-agent systems.

5. CONCLUSIONS

The adaptive algorithm for linear SISO plants with any relative degree under parametric uncertainties and input delay is proposed. The new parameterization of the plant equation is considered. The proposed algorithm ensures small difference between plant output and reference model output in the steady state. The modeling results illustrate the effectiveness of the proposed algorithm.

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APPENDIX A.

Proof of Theorem. Rewrite (23) as follows

$$\alpha(t-h) = \alpha(t) + \rho_1 \int_{t-h}^{t} e(s) w(s-h) ds,$$
$$\beta(t-h, g) = \beta(t, g) + \rho_2 \int_{t-h}^{t} e(s) f(s+g-h) ds. \tag{27}$$

Taking into account (21), (22) and (27), rewrite the equation of the closed loop system in the form

$$\begin{aligned} e(t) = \Bigg[\Bigg((\alpha(t)-\alpha_0)^{\mathrm{T}} + \rho_1 \int_{t-h}^{t} e(s) w^{\mathrm{T}}(s-h) ds \Bigg) w(t-h) + \\ + \int_{-h}^{0} (\beta(t, g) - \beta_0(g))^{\mathrm{T}} f(t+g-h) dg \\ + \rho_2 \int_{-h}^{0} \int_{t-h}^{t} e(s) f^{\mathrm{T}}(s+g-h) ds f(t+g-h) dg \Bigg] k \\ + \int_{-h}^{0} \gamma_0^{\mathrm{T}} \frac{R(p)}{R_m(p)} \mu_2^{\gamma-1} L \eta(t-h+g) dg \\ + k \frac{R(p)}{R_m(p)} \mu_2^{\gamma-1} L \eta(t-h), \quad \mu_1 \dot{\eta}(t) = G\eta(t) + \mu_2 q \dot{v}(t). \end{aligned} \tag{28}$$

To analyze system (28) the following Lemma (Furtat et. al 2014) is needed.

Lemma. Let the system be described by the following differential equation

$$\dot{x} = f(x, \mu_1, \mu_2, t), \tag{29}$$

where $x \in R^{s_1}$, $\mu = \mathrm{col}(\mu_1, \mu_2) \in R^{s_2}$, $f(x, \mu_1, \mu_2, t)$ is Lipchitz continuous function in x and bounded on t. Let (29) have a bounded closed set of attraction $\Omega = \{x \mid P(x) \le C\}$ for $\mu_2 = 0$, where $P(x)$ is piecewise-smooth, positive definite function in R^{s_1}. In addition let there exist some numbers $C_1 > 0$ and $\overline{\mu}_1 > 0$ such that the following condition holds

$$\sup_{|\mu_1| \le \overline{\mu}_1} \left[\left\langle \left[\frac{\partial P(x)}{\partial x} \right]^{\mathrm{T}}, f(x, \mu_1, 0, t) \right\rangle \middle| \quad P(x) = C \right] \le -C_1.$$

Then there exists $\mu_0 > 0$ such that the system (29) has the same set of attraction Ω for $\mu_2 \le \mu_0$.

Let us check conditions of Lemma. Consider system (28) for $\mu_2 = 0$. Let $P(x) = V$, where V is Lyaponov-Krasovskii functional in the form

$$\begin{aligned} V = \frac{k}{2\rho_1} (\alpha(t) - \alpha_0)^{\mathrm{T}} (\alpha(t) - \alpha_0) + \eta^T(t) H \eta^T(t) \\ + \frac{k}{2\rho_2} \int_{-h}^{0} (\beta(t, g) - \beta_0(g))^{\mathrm{T}} (\beta(t, g) - \beta_0(g)) dg. \\ + \int_{-h}^{0} ds \int_{t+s}^{t} \dot{\alpha}^T(z) \dot{\alpha}(z) dz \\ + \int_{-h}^{0} \int_{t+s}^{t} \int_{-h}^{0} \dot{\beta}^T(\tau, g) \dot{\beta}(\tau, g) d\tau ds dg, \end{aligned} \tag{30}$$

where $H = H^{\mathrm{T}} > 0$ is the solution of the equation $H^{\mathrm{T}}G + GH = -Q$, $Q = Q^{\mathrm{T}} > 0$, $H = H^{\mathrm{T}} > 0$.

Differentiating (30) along trajectories of (23), and substituting the right hand side of (28) into (29), find $\dot{V}$ as follows

$$\begin{aligned} \dot{V} = -e^2(t) \\ + k\rho_1 e(t) \int_{t-h}^{t} e(s) w^{\mathrm{T}}(s-h) ds \cdot w(t-h) \\ + k\rho_2 e(t) \int_{-h}^{0} \int_{t-h}^{t} e(s) f^{\mathrm{T}}(s+g-h) ds \cdot f(t+g-h) dg \\ + h\rho_1^2 e^2(t) |w(t-h)|^2 - \int_{t-h}^{t} \dot{\alpha}^{\mathrm{T}}(s) \dot{\alpha}(s) ds \\ + h\rho_1^2 e^2(t) \int_{-h}^{0} |f(t+g-h)|^2 dg \\ - \int_{-h}^{0} \int_{-h}^{0} \dot{\beta}^{\mathrm{T}}(t+s, g) \dot{\beta}(t+s, g) ds dg - \mu_1^{-1} \eta^{\mathrm{T}}(t) Q \eta(t). \end{aligned} \tag{31}$$

Consider the following upper bounds:

$$k\rho_1 e(t) \int_{t-h}^{t} e(s) w^T(s-h) ds w(t-h)$$
$$\le k\rho_1 \left[\chi e^2(t) |w(t-h)|^2 + \rho_1^{-2} \chi^{-1} \int_{t-h}^{t} \dot{\alpha}^{\mathrm{T}}(s) ds \int_{t-h}^{t} \dot{\alpha}(s) ds \right];$$
$$k\rho_2 e(t) \int_{-h}^{0} \int_{t-h}^{t} e(s) f^T(s+g-h) ds f(t+g-h) dg$$
$$\le k\rho_2 \left[\chi e^2(t) \int_{-h}^{0} |f(t+g-h)|^2 dg \right.$$
$$\left. + \rho_2^{-2} \chi^{-1} \int_{-h}^{0} \int_{-h}^{0} \dot{\beta}^{\mathrm{T}}(t+s, g) ds dg \int_{-h}^{0} \int_{-h}^{0} \beta(t+s, g) ds dg \right],$$
$$- \int_{t-h}^{t} \dot{\alpha}^{\mathrm{T}}(s) \dot{\alpha}(s) ds \le -h^{-1} \int_{t-h}^{t} \dot{\alpha}^{\mathrm{T}}(s) ds \int_{t-h}^{t} \dot{\alpha}(s) ds,$$
$$- \int_{-h}^{0} \int_{-h}^{0} \dot{\beta}^{\mathrm{T}}(t+s, g) \dot{\beta}(t+s, g) ds dg$$

$$\le -2h^{-2}\int_{-h}^{0}\int_{-h}^{0}\dot\beta^{\mathrm T}(t+s,g)dsdg\int_{-h}^{0}\int_{-h}^{0}\dot\beta(t+s,g)dsdg,$$

where $\chi > 0$.

Taking into account upper bounds, rewrite (31) as follows

$$\begin{aligned}\dot V \le &-q_1(t)e^2(t)-q_2\int_{t-h}^{t}\dot\alpha^{\mathrm T}(s)\dot\alpha(s)ds\\ &-q_3\int_{-h}^{0}\int_{-h}^{0}\dot\beta^{\mathrm T}(t+s,z)\beta(t+s,z)ds-\mu_1^{-1}\eta^T(t)Q\eta(t),\end{aligned}\tag{32}$$

where

$$\begin{aligned}q_1(t)=&1-\left(k\rho_1\chi+h\rho_1^2\right)\left|w(t-h)\right|^2\\ &-\left(k\rho_2\chi+h\rho_2^2\right)\int_{-h}^{0}\left|f(t+g-h)\right|dg,\\ q_2=&\,h^{-1}-\rho_1^{-1}\chi^{-1},\quad q_3=2h^{-2}-\rho_2^{-1}\chi^{-1}.\end{aligned}\tag{33}$$

It is obvious, that there exist ρ_1, ρ_2 and χ such that $q_2 > 0$, $q_3 > 0$. Therefore, we have $\lim_{t\to\infty} e(t) = 0$ when $q_1(t) > 0$.

However, the value of $q_1(t)$ depends on $w(t)$ and $f(t)$. The signals $w(t)$ and $f(t)$ depend on bounded signal $r(t)$ and functions $\theta_1(t)$ and $\theta_2(t)$ which depend on the control law $u(t)$. The proof of boundedness of vectors $w(t)$ and $f(t)$ is based on results of (Niculescu, 2001; Niculescu and Annaswamy 2003).

Consider the vectors $w(t)$ and $f(t)$ on the interval $[t_0 - h, t_0)$, where t_0 is an initial time. Obviously, that $\sup_{\sigma_0\in[t_0-h,t_0]}\left\|u(\sigma_0)\right\|^2 \le l_0$, where $l_0 > 0$. Then $\sup_{\sigma_0\in[t_0-h,t_0]}\left\|w(\sigma_0)\right\|^2 \le z_{01}$ and $\sup_{\sigma_0\in[t_0-h,t_0]}\left\|\int_{-h}^{0}\left|f(\sigma_0+g-h)\right|dg\right\|^2 \le z_{02}$, where $z_{01} > 0$ and $z_{02} > 0$. Then, there exist $\rho_{01} > 0$ and $\rho_{02} > 0$ such that for $\rho_1 = \rho_{01}$ and $\rho_2 = \rho_{02}$ the following inequality holds

$$1-\left(k\rho_{01}\chi+h\rho_{01}^2\right)z_{01}-\left(k\rho_{02}\chi+h\rho_{02}^2\right)z_{02} > 0 \text{ for } [t_0-h,\, t_0).$$

Therefore, Lyapunov-Krasovskii functional (30) is non-increasing on the time-interval $[t_0 - h, t_0)$.

Consider the vectors $w(t)$ and $f(t)$ on the interval $[t_0 + (i - 1)h, t_0 + ih)$, $i \in N$, N is a set of natural numbers. Obviously, that $\sup_{\sigma_{i+1}\in[t_0+(i-1)h,t_0+ih]}\left\|u(\sigma_i)\right\|^2 \le l_i$, where $l_i > 0$. Then $\sup_{\sigma_{i+1}\in[t_0+(i-1)h,t_0+(ih]}\left\|w(\sigma_i)\right\|^2 \le z_{i,1}$ and $\sup_{\sigma_{i+1}\in[t_0+(i-1)h,t_0+ih]}\left\|\int_{-h}^{0}\left|f(\sigma_i+g-h)\right|dg\right\|^2 \le z_{i,2}$, where $z_{i,1} > 0$ and $z_{i,2} > 0$. Then, there exist $\rho_{i,1} > 0$ and $\rho_{i,2} > 0$ such that for $\rho_1 = \rho_{i,1}$ and $\rho_2 = \rho_{i,2}$ the following inequality holds

$$1-\left(k\rho_{i,1}\chi+h\rho_{i,1}^2\right)z_{i,1}-\left(k\rho_{i,2}\chi+h\rho_{i,2}^2\right)z_{i,2} > 0$$

$$\text{for } [t_0+(i-1)h,\, t_0+ih),\ i\in N.$$

Therefore, Lyapunov-Krasovskii functional (30) is non-increasing on the time-interval $[t_0 + (i - 1)h, t_0 + ih)$, $i \in N$. Choosing parameters as $\rho_1 = \min\{\rho_{01}, \rho_{11}, \rho_{i,1}\}$ and $\rho_2 = \min\{\rho_{02}, \rho_{12}, \rho_{i,2}\}$, where $i \in N$ we can ensure that functional (30) is non-increasing for any interval time. Therefore, system (28) is asymptotically stable and all signals are bounded in the closed loop system. Then, let us denote

$$\begin{aligned}\sup_t\Bigg|k\int_{-h}^{0}e^{-Ag}Dk\frac{R(p)}{R_m(p)}\mu_0^{\gamma-1}L\eta(t-h+g)dg\\ +k\mu_0^{\gamma-1}L\eta(t-h)\Bigg|\le k_1,\\ \sup_t|\dot v(t)|\le k_2,\ k\le k_3,\end{aligned}$$

where $k_1 > 0$, $k_2 > 0$, $k_3 > 0$. Moreover, the value k_1 can be reduced by decreasing the values μ.

However, asymptotic stability of system (28) for $\mu_2 \neq 0$ does not follow from asymptotic stability of the solution of system (28) for $\mu_2 \neq 0$. Therefore, consider (28) for $\mu_1 = \mu_2 = \mu_0$. Taking into account result (32), taking the derivative of (30) along the trajectories of (28), we obtain

$$\begin{aligned}\dot V \le &-q_1(t)e^2(t)-q_2\int_{t-h}^{t}\dot\alpha^{\mathrm T}(s)\dot\alpha(s)ds\\ &-q_3\int_{-h}^{0}\int_{-h}^{0}\dot\beta^{\mathrm T}(t+s,z)\beta(t+s,z)ds\\ &-kk_1e(t)-\mu_0^{-1}\eta^{\mathrm T}(t)Q\eta(t)+2\eta^{\mathrm T}(t)Hqk_2.\end{aligned}\tag{34}$$

Consider the following upper bounds

$$-kk_1e(t)\le 0.5\chi^{-1}k_3e^2(t)+0.5\chi k_3k_1^2,$$

$$2\eta^{\mathrm T}(t)Hqk_2\le\chi^{-1}\eta^{\mathrm T}(t)Hqq^{\mathrm T}H\eta(t)+\chi k_2^2,$$

and rewrite (34) in the form

$$\begin{aligned}\dot V \le &-\left[q_1(t)-0.5k_3\chi^{-1}\right]e^2(t)-q_2\int_{t-h}^{t}\dot\alpha^{\mathrm T}(s)\dot\alpha(s)ds\\ &-q_3\int_{-h}^{0}\int_{-h}^{0}\dot\beta^{\mathrm T}(t+s,z)\beta(t+s,z)ds\\ &-\mu_0^{-1}\eta^T(t)\left[Q-\mu_0\chi^{-1}Hqq^TH\right]\eta(t)+\psi,\end{aligned}$$

where $\psi = \chi\left(0.5k_3k_1^2+k_2^2\right)$. There exist ρ_1, ρ_2 and χ such that $q_1(t)-0.5k_3\chi^{-1} > 0$, $q_2 > 0$, $q_3 > 0$ and $Q-\mu_0\chi^{-1}Hqq^TH > 0$. Therefore, Lyapunov-Krasovskii functional (30) is non-increasing. Then, goal (3) holds and all signals in the closed loop system are bounded. Theorem is proved.