

11.8 Power Series

A **power series** is a series of the form

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \dots$$

Here $\{c_n\}$ is the sequence of coefficients and a is the **center** or **base point** of the power series.

When we plug a value for x into a power series, we get a “normal” series.

Example: $\sum_{n=0}^{\infty} \frac{n}{2^n}(x-3)^n$ ← A power series with base point $a=3$
and coefficients $c_n = \frac{n}{2^n}$

If we plug in $x=2$, we get $\sum_{n=0}^{\infty} \frac{n}{2^n}(-1)^n$ ← A “normal” series
(converges by AST)

A power series is a FUNCTION of x (plugging in a value for x gives exactly one output... except when the series diverges, which is why we define the interval of convergence.)

Definition: The **interval of convergence** (IOC) of a power series is the set of all points, x , for which the power series converges.

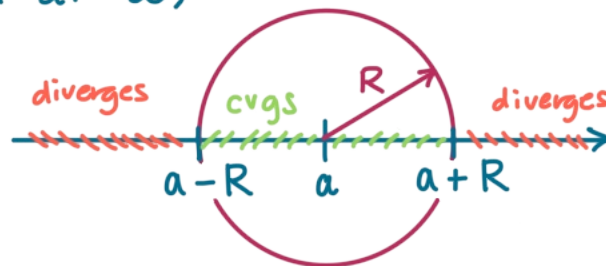
Facts about the Interval of Convergence:

- If a power series converges at two separate points, it also converges at all points between them. In other words, the interval of convergence will be an INTERVAL (unless it's a single point).
- The base point is ALWAYS in the interval of convergence. In other words, a power series always converges at its base point.
- The interval of convergence is always SYMMETRIC about the base point, which leads us to define something called the radius of convergence.

Definition: The **radius of convergence** of a power series $\sum c_n(x-a)^n$ is a number $R \geq 0$ such that

- if $|x-a| < R$, the series converges at x
- if $|x-a| > R$, the series diverges at x

Imagine a circle of radius R centered at a ,
that intersects the number line:



Facts about the Radius of Convergence:

- We use the Ratio Test to find the radius of convergence.
- The radius of convergence does not tell you what happens when $|x - a| = R$ (i.e. when x is one of the endpoints of the IOC)
- If $R = \infty$, that means the power series converges at EVERY real number, x , so the interval of convergence is $(-\infty, \infty)$.
- If $R = 0$, that means the power series converges ONLY at the base point.