

PRACTICE EXAM 2B solutions

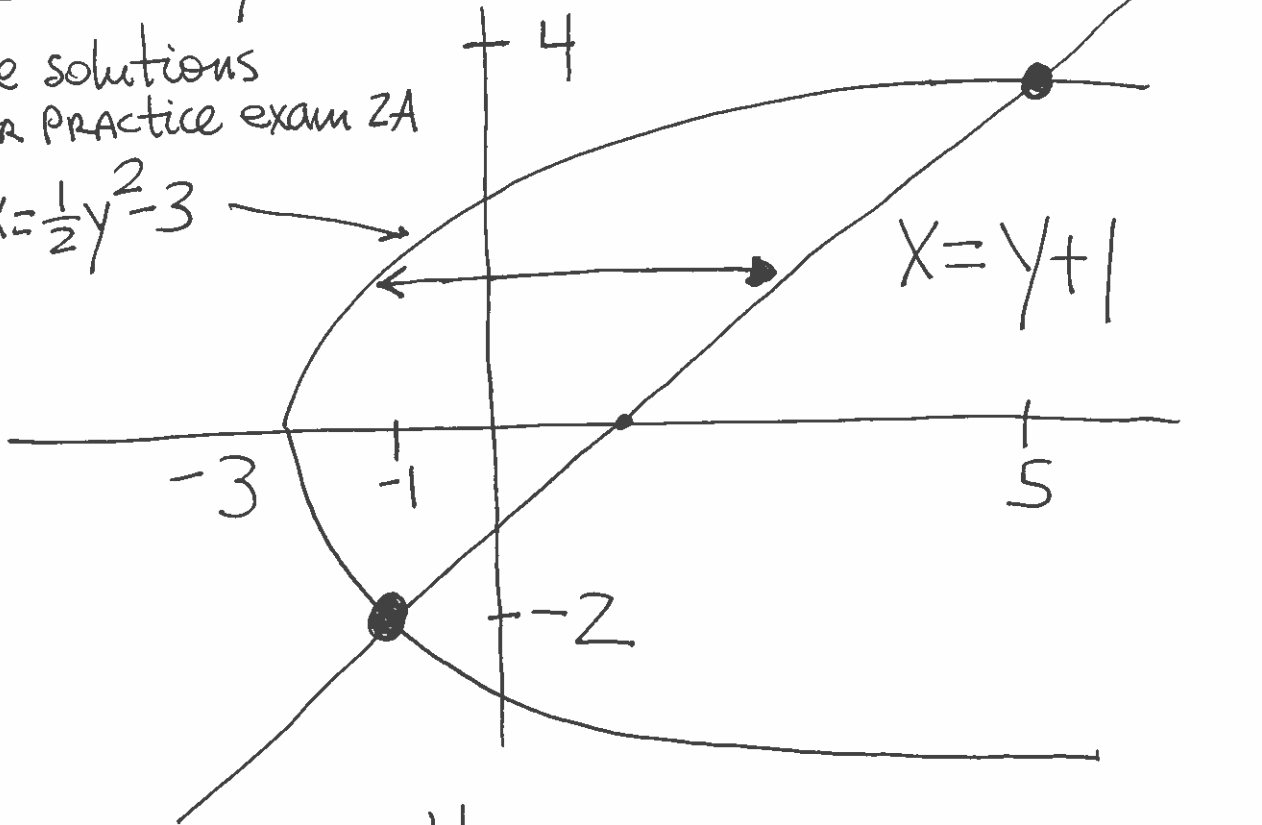
#1

$$y^2 = 2x + 6 \quad y = x - 1$$

see solutions
for PRACTICE exam 2A

$$x = \frac{1}{2}y^2 - 3$$

$$x = y + 1$$



$$\text{Area} = \int_{-2}^4 \left[(y+1) - \left(\frac{1}{2}y^2 - 3 \right) \right] dy =$$

$$\int_{-2}^4 \left[-\frac{1}{2}y^2 + y + 4 \right] dy$$

$$\#2 \quad \frac{x^3 + 5x^2 + 12x + 15}{(2x+5)^2 (2x^2+1)^2} = \frac{A}{2x+5} + \frac{B}{(2x+5)^2} +$$

$$\frac{Cx+D}{2x^2+1} + \frac{Ex+F}{(2x^2+1)^2}$$

$$\#3 \quad x = \sqrt{5} \sin \theta \quad dx = \sqrt{5} \cos \theta d\theta$$

$$5 - x^2 = 5 - 5 \sin^2 \theta = 5(1 - \sin^2 \theta) = 5 \cos^2 \theta$$

$$\int \frac{5 \sin^2 \theta}{\sqrt{5 \cos^2 \theta}} \sqrt{5} \cos \theta d\theta =$$

$$\int \frac{5 \sin^2 \theta}{\sqrt{5} \cos \theta} \sqrt{5} \cos \theta d\theta = 5 \int \sin^2 \theta d\theta$$

$$\#4 \quad 2x^2 - 2x + 5 = A(x^2 + 4) + (Bx + C)(x - 1) \quad \left\{ \begin{array}{l} \text{CHECK} \\ 4 - C \stackrel{?}{=} 5 \\ 4 - (-1) = 5 \\ 5 = 5 \checkmark \end{array} \right.$$

$$\text{If } x=1 \quad 2-2+5=5A \quad \boxed{A=1}$$

$$2x^2 - 2x + 5 = x^2 + 4 + Bx^2 - Bx + Cx - C$$

$$2x^2 - 2x + 5 = (1+B)x^2 + (C-B)x + (4-C)$$

$$2 = 1+B \quad \boxed{B=1} \quad C-B = -2 \quad C-1 = -2 \quad \boxed{C=-1}$$

#5

$$\int \sqrt{x^2 - 9} \, dx$$

$$x = 3 \sec \theta$$

$$dx = 3 \sec \theta \tan \theta \, d\theta$$

$$x^2 - 9 = 9 \sec^2 \theta - 9 =$$

$$9(\sec^2 \theta - 1) = 9 \tan^2 \theta$$

$$\int \sqrt{x^2 - 9} \, dx \rightarrow \int \sqrt{9 \tan^2 \theta} \, 3 \sec \theta \tan \theta \, d\theta =$$

$$\int 3 \tan \theta \cdot 3 \sec \theta \tan \theta \, d\theta = 9 \int \sec \theta \tan^2 \theta \, d\theta =$$

$$9 \int \sec \theta (\sec^2 \theta - 1) \, d\theta = 9 \int (\sec^3 \theta - \sec \theta) \, d\theta =$$

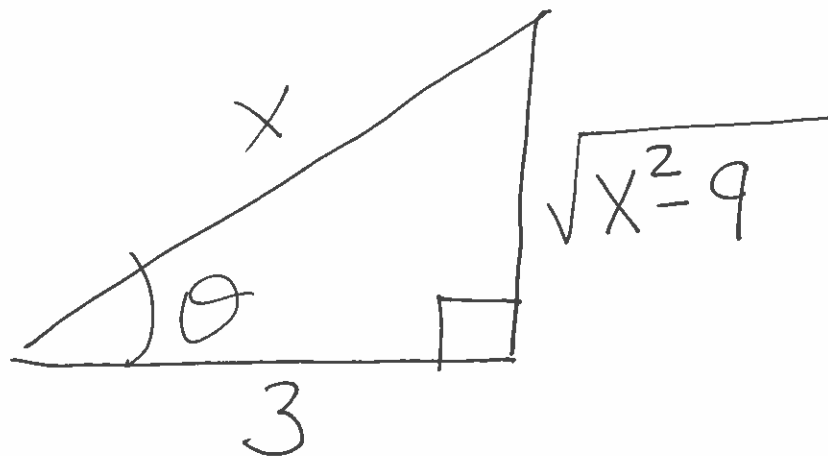
$$\int \sec^3 \theta \, d\theta = \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta|$$

$$\int \sec \theta \, d\theta = \ln |\sec \theta + \tan \theta|$$

$$9 \int \sec^3 \theta \, d\theta - 9 \int \sec \theta \, d\theta = \frac{9}{2} \sec \theta \tan \theta - \frac{9}{2} \ln |\sec \theta + \tan \theta| +$$

$$9 \ln |\sec \theta + \tan \theta| = \frac{9}{2} \sec \theta \tan \theta - \frac{9}{2} \ln |\sec \theta + \tan \theta|$$

$$X = 3 \sec \theta \quad \sec \theta = X/3$$



$$\int \sqrt{x^2 - 9} \, dx = \frac{9}{2} \frac{x}{3} \frac{\sqrt{x^2 - 9}}{3} - \frac{9}{2} \ln \left| \frac{x}{3} + \frac{\sqrt{x^2 - 9}}{3} \right|$$

$$\int \sqrt{x^2 - 9} \, dx = \frac{1}{2} x \sqrt{x^2 - 9} - \frac{9}{2} \ln \left| \frac{x}{3} + \frac{\sqrt{x^2 - 9}}{3} \right| + C$$

$$\text{\#6} \quad \frac{16}{(x^2 - 4)(x - 2)} = \frac{16}{(x - 2)(x + 2)(x - 2)} = \frac{16}{(x - 2)^2(x + 2)} =$$

$$\frac{A}{x - 2} + \frac{B}{(x - 2)^2} + \frac{C}{x + 2}$$

$$16 = A(x - 2)(x + 2) + B(x + 2) + C(x - 2)^2$$

$$x = -2 \quad 16 = 16C \quad \boxed{C = 1}$$

$$x = 2 \quad 16 = 4B \quad \boxed{B = 4}$$

$$16 = A(x^2 - 4) + 4x + 8 + x^2 - 4x + 4$$

$$16 = Ax^2 - 4A + 12 + x^2$$

$$(A+1)x^2 + 12 - 4A = 16 \quad ?$$

$$A+1=0 \quad \boxed{A=-1} \quad \text{CHECK: } 12-4A \stackrel{?}{=} 16$$

$$12-4(-1) = 12+4 = 16 \quad \checkmark$$

$$\int \frac{16}{(x^2-4)(x+2)} dx = -\int \frac{1}{x-2} dx + 4 \int \frac{1}{(x-2)^2} dx + \int \frac{1}{x+2} dx =$$

$$-\ln|x-2| - \frac{4}{x-2} + \ln|x+2| + C$$

#7

$$\int_e^{\infty} \frac{\ln x}{x} dx$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$x=e \quad u = \ln e = 1$$

$$x \rightarrow \infty \quad u = \ln x \rightarrow \infty$$

$$\int_1^{\infty} u du = \lim_{t \rightarrow \infty} \int_1^t u du = \lim_{t \rightarrow \infty} \left. \frac{1}{2} u^2 \right|_1^t =$$

$$\lim_{t \rightarrow \infty} \left(\frac{1}{2} t^2 - \frac{1}{2} \right) \rightarrow \infty$$

$$\int_e^{\infty} \frac{\ln x}{x} dx$$

DIVERGES

#8 $\frac{\pi}{4} \leq \text{Arctan } x \leq \frac{\pi}{2} \text{ for } x \geq 1$

$$3 + \frac{\pi}{4} \leq 3 + \text{Arctan } x \leq 3 + \frac{\pi}{2}$$

$$\frac{3 + \frac{\pi}{4}}{\sqrt{x}} \leq \frac{3 + \text{Arctan } x}{\sqrt{x}} \leq \frac{3 + \frac{\pi}{2}}{\sqrt{x}}$$

$$(3 + \frac{\pi}{4}) \int_1^{\infty} \frac{1}{x^{1/2}} dx \leq \int_1^{\infty} \frac{3 + \text{Arctan } x}{\sqrt{x}} dx \leq (3 + \frac{\pi}{2}) \int_1^{\infty} \frac{1}{x^{1/2}} dx$$

$\int_1^{\infty} \frac{1}{x^{1/2}} dx$ Diverges $p = \frac{1}{2}$ integral

By the left side of the inequality

$$\infty \leq \int_1^{\infty} \frac{3 + \text{Arctan } x}{\sqrt{x}} dx$$

Diverges
by
Direct Comparison
Test.