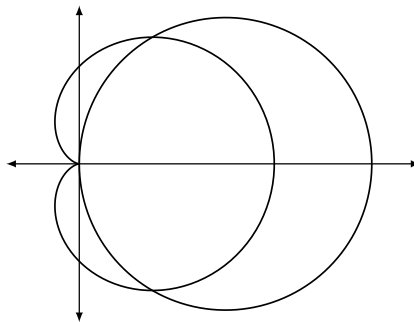


Purpose: To practice finding the area between polar curves and finding correct bounds for a given region.

Warm-up Question: Write down the formula for the area enclosed by a polar curve $r = f(\theta)$ for $a \leq \theta \leq b$ and explain how it comes from the formula for the area of a sector of a circle.

The rest of the questions in this handout will have to do with the polar curves $r = 3\cos\theta$ and $r = 1 + \cos\theta$, graphed below.

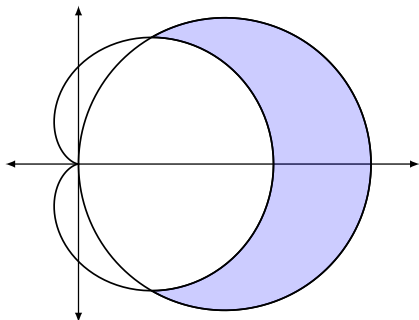
1. Which curve is $r = 3\cos\theta$ and which is $r = 1 + \cos\theta$? Label them on the graph.



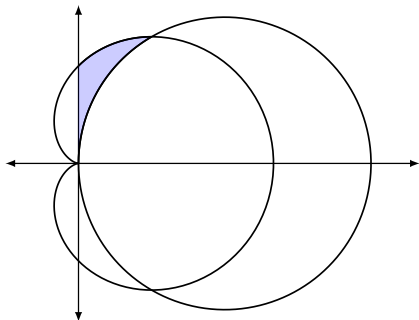
2. Find the intersection points of the two curves for θ in $[0, 2\pi]$. Can you explain what's going on at the origin?

3. Match the shaded region with an integral that represents the area.

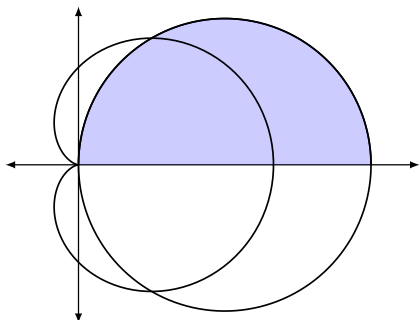
I.



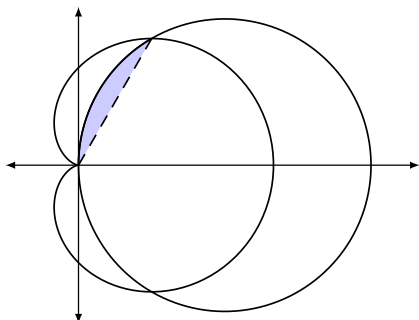
II.



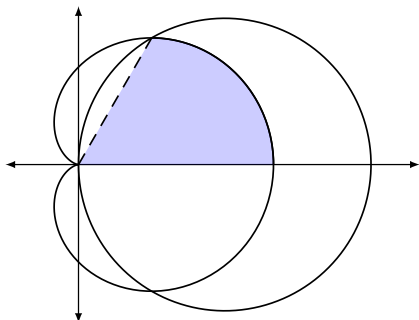
III.



IV.



V.



(a) $\int_0^{\pi/2} \frac{1}{2} \left((1 + \cos \theta)^2 - (3 \cos \theta)^2 \right) d\theta$

(b) $\int_{-\pi/3}^{\pi/3} \frac{1}{2} \left((1 + \cos \theta)^2 - (3 \cos \theta)^2 \right) d\theta$

(c) $\int_{\pi/3}^{\pi/2} \frac{1}{2} \left((1 + \cos \theta)^2 - (3 \cos \theta)^2 \right) d\theta$

(d) $\int_{\pi/3}^{5\pi/3} \frac{1}{2} \left((3 \cos \theta)^2 - (1 + \cos \theta)^2 \right) d\theta$

(e) $\int_{-\pi/3}^{\pi/3} \frac{1}{2} \left((3 \cos \theta)^2 - (1 + \cos \theta)^2 \right) d\theta$

(f) $\int_{\pi/3}^{\pi/2} \frac{1}{2} \left((3 \cos \theta)^2 - (1 + \cos \theta)^2 \right) d\theta$

(g) $\int_0^{\pi} \frac{1}{2} (3 \cos \theta)^2 d\theta$

(h) $\int_0^{\pi/2} \frac{1}{2} (3 \cos \theta)^2 d\theta$

(i) $\int_0^{\pi/3} \frac{1}{2} (3 \cos \theta)^2 d\theta$

(j) $\int_{\pi/3}^{\pi/2} \frac{1}{2} (3 \cos \theta)^2 d\theta$

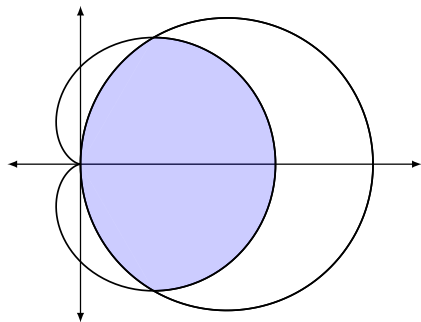
(k) $\int_0^{\pi/3} \frac{1}{2} (1 + \cos \theta)^2 d\theta$

(l) $\int_{\pi/3}^{\pi/2} \frac{1}{2} (1 + \cos \theta)^2 d\theta$

(m) $\int_0^{\pi} \frac{1}{2} (1 + \cos \theta)^2 d\theta$

4. The area of the shaded regions below cannot be represented with a single integral. (Can you see why?) Write down a **sum** of integrals that represents the shaded area for each figure.

(a)



(b)

