

(b) $C(t) = \frac{1}{t} \left[A + \int_0^t f(s) ds \right] = \frac{A + F(t)}{t}$ represents the average expenditure per unit of t during the interval $[0, t]$,

assuming that there has been only one overhaul during that time period. The company wants to minimize average expenditure.

(c) $C(t) = \frac{1}{t} \left[A + \int_0^t f(s) ds \right]$. Using FTC1, we have $C'(t) = -\frac{1}{t^2} \left[A + \int_0^t f(s) ds \right] + \frac{1}{t} f(t)$.

$$C'(t) = 0 \Rightarrow t f(t) = A + \int_0^t f(s) ds \Rightarrow f(t) = \frac{1}{t} \left[A + \int_0^t f(s) ds \right] = C(t).$$

5.4 Indefinite Integrals and the Net Change Theorem

- $\frac{d}{dx} [x \ln x - x + C] = x \cdot \frac{1}{x} + (\ln x) \cdot 1 - 1 + 0 = 1 + \ln x - 1 = \ln x$
- $\frac{d}{dx} (\tan x - x + C) = \sec^2 x - 1 + 0 = \tan^2 x$
- $$\begin{aligned} \frac{d}{dx} \left[-\frac{\sqrt{1+x^2}}{x} + C \right] &= \frac{d}{dx} \left[-\frac{(1+x^2)^{1/2}}{x} + C \right] = -\frac{x \cdot \frac{1}{2}(1+x^2)^{-1/2}(2x) - (1+x^2)^{1/2} \cdot 1}{(x)^2} + 0 \\ &= -\frac{(1+x^2)^{-1/2} [x^2 - (1+x^2)]}{x^2} = -\frac{-1}{(1+x^2)^{1/2} x^2} = \frac{1}{x^2 \sqrt{1+x^2}} \end{aligned}$$
- $$\begin{aligned} \frac{d}{dx} \left[\frac{2}{15b^2} (3bx - 2a)(a + bx)^{3/2} + C \right] &= \frac{2}{15b^2} \left[(3bx - 2a) \frac{3}{2} (a + bx)^{1/2} (b) + (a + bx)^{3/2} (3b) + 0 \right] \\ &= \frac{2}{15b^2} (3b)(a + bx)^{1/2} \left[(3bx - 2a) \frac{1}{2} + (a + bx) \right] \\ &= \frac{2}{5b} (a + bx)^{1/2} \left(\frac{5}{2} bx \right) = x \sqrt{a + bx} \end{aligned}$$
- $\int (3x^2 + 4x + 1) dx = 3 \cdot \frac{1}{3} x^3 + 4 \cdot \frac{1}{2} x^2 + x + C = x^3 + 2x^2 + x + C$
- $\int (5 + 2\sqrt{x}) dx = \int (5 + 2x^{1/2}) dx = 5x + 2 \cdot \frac{2}{3} x^{3/2} + C = 5x + \frac{4}{3} x^{3/2} + C$
- $\int (x + \cos x) dx = \frac{1}{2} x^2 + \sin x + C$
- $\int \left(\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \right) dx = \int (x^{1/3} + x^{-1/3}) dx = \frac{3}{4} x^{4/3} + \frac{3}{2} x^{2/3} + C$
- $\int (x^{1.3} + 7x^{2.5}) dx = \frac{1}{2.3} x^{2.3} + \frac{7}{3.5} x^{3.5} + C = \frac{1}{2.3} x^{2.3} + 2x^{3.5} + C$
- $\int \sqrt[4]{x^5} dx = \int x^{5/4} dx = \frac{4}{9} x^{9/4} + C$
- $\int \left(5 + \frac{2}{3} x^2 + \frac{3}{4} x^3 \right) dx = 5x + \frac{2}{3} \cdot \frac{1}{3} x^3 + \frac{3}{4} \cdot \frac{1}{4} x^4 + C = 5x + \frac{2}{9} x^3 + \frac{3}{16} x^4 + C$
- $\int (u^6 - 2u^5 - u^3 + \frac{2}{7}) du = \frac{1}{7} u^7 - 2 \cdot \frac{1}{6} u^6 - \frac{1}{4} u^4 + \frac{2}{7} u + C = \frac{1}{7} u^7 - \frac{1}{3} u^6 - \frac{1}{4} u^4 + \frac{2}{7} u + C$

$$13. \int (u+4)(2u+1) du = \int (2u^2 + 9u + 4) du = 2 \frac{u^3}{3} + 9 \frac{u^2}{2} + 4u + C = \frac{2}{3}u^3 + \frac{9}{2}u^2 + 4u + C$$

$$14. \int \sqrt{t}(t^2 + 3t + 2) dt = \int t^{1/2}(t^2 + 3t + 2) dt = \int (t^{5/2} + 3t^{3/2} + 2t^{1/2}) dt \\ = \frac{2}{7}t^{7/2} + 3 \cdot \frac{2}{5}t^{5/2} + 2 \cdot \frac{2}{3}t^{3/2} + C = \frac{2}{7}t^{7/2} + \frac{6}{5}t^{5/2} + \frac{4}{3}t^{3/2} + C$$

$$15. \int \frac{1 + \sqrt{x} + x}{x} dx = \int \left(\frac{1}{x} + \frac{\sqrt{x}}{x} + \frac{x}{x} \right) dx = \int \left(\frac{1}{x} + x^{-1/2} + 1 \right) dx \\ = \ln |x| + 2x^{1/2} + x + C = \ln |x| + 2\sqrt{x} + x + C$$

$$16. \int \left(x^2 + 1 + \frac{1}{x^2 + 1} \right) dx = \frac{x^3}{3} + x + \tan^{-1} x + C$$

$$17. \int \left(e^x + \frac{1}{x} \right) dx = e^x + \ln |x| + C$$

$$18. \int (2 + 3^x) dx = 2x + \frac{3^x}{\ln 3} + C$$

$$19. \int (\sin x + \sinh x) dx = -\cos x + \cosh x + C$$

$$20. \int \left(\frac{1+r}{r} \right)^2 dr = \int \frac{1+2r+r^2}{r^2} dr = \int (r^{-2} + 2r^{-1} + 1) dr = -r^{-1} + 2 \ln |r| + r + C = -\frac{1}{r} + 2 \ln |r| + r + C$$

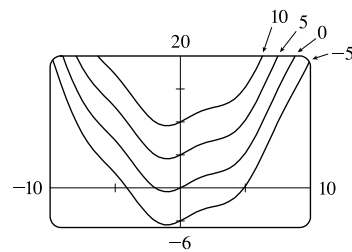
$$21. \int (2 + \tan^2 \theta) d\theta = \int [2 + (\sec^2 \theta - 1)] d\theta = \int (1 + \sec^2 \theta) d\theta = \theta + \tan \theta + C$$

$$22. \int \sec t (\sec t + \tan t) dt = \int (\sec^2 t + \sec t \tan t) dt = \tan t + \sec t + C$$

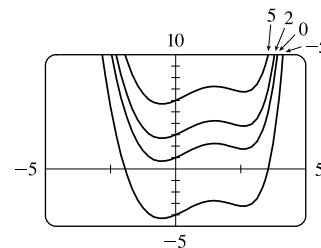
$$23. \int 3 \csc^2 t dt = -3 \cot t + C$$

$$24. \int \frac{\sin 2x}{\sin x} dx = \int \frac{2 \sin x \cos x}{\sin x} dx = \int 2 \cos x dx = 2 \sin x + C$$

25. $\int (\cos x + \frac{1}{2}x) dx = \sin x + \frac{1}{4}x^2 + C$. The members of the family in the figure correspond to $C = -5, 0, 5$, and 10 .



26. $\int (e^x - 2x^2) dx = e^x - \frac{2}{3}x^3 + C$. The members of the family in the figure correspond to $C = -5, 0, 2$, and 5 .



27. $\int_{-2}^3 (x^2 - 3) dx = \left[\frac{1}{3}x^3 - 3x\right]_{-2}^3 = (9 - 9) - \left(-\frac{8}{3} + 6\right) = \frac{8}{3} - \frac{18}{3} = -\frac{10}{3}$
28. $\int_1^2 (4x^3 - 3x^2 + 2x) dx = \left[x^4 - x^3 + x^2\right]_1^2 = (16 - 8 + 4) - (1 - 1 + 1) = 12 - 1 = 11$
29. $\int_1^4 (8t^3 - 6t^{-2}) dt = \left[2t^4 + \frac{6}{t}\right]_1^4 = \left(2 \cdot 4^4 + \frac{6}{4}\right) - \left(2 \cdot 1^4 + \frac{6}{1}\right) = \left(512 + \frac{3}{2}\right) - (2 + 6) = \frac{1011}{2} = 505.5$
30. $\int_0^8 \left(\frac{1}{8} + \frac{1}{2}w + \frac{1}{3}w^{1/3}\right) dw = \left[\frac{1}{8}w + \frac{1}{4}w^2 + \frac{1}{4}w^{4/3}\right]_0^8 = (1 + 16 + 4) - 0 = 21$
31. $\int_0^2 (2x - 3)(4x^2 + 1) dx = \int_0^2 (8x^3 - 12x^2 + 2x - 3) dx = \left[2x^4 - 4x^3 + x^2 - 3x\right]_0^2 = (32 - 32 + 4 - 6) - 0 = -2$
32. $\int_1^2 \left(\frac{1}{x^2} - \frac{4}{x^3}\right) dx = \int_1^2 (x^{-2} - 4x^{-3}) dx = \left[\frac{x^{-1}}{-1} - \frac{4x^{-2}}{-2}\right]_1^2 = \left[-\frac{1}{x} + \frac{2}{x^2}\right]_1^2 = \left(-\frac{1}{2} + \frac{1}{2}\right) - (-1 + 2) = -1$
33. $\int_1^3 \left(\frac{3x^2 + 4x + 1}{x}\right) dx = \int_1^3 \left(3x + 4 + \frac{1}{x}\right) dx = \left[\frac{3}{2}x^2 + 4x + \ln|x|\right]_1^3 = \left(\frac{27}{2} + 12 + \ln 3\right) - \left(\frac{3}{2} + 4 + \ln 1\right) = 20 + \ln 3$
34. $\int_{-1}^1 t(1 - t)^2 dt = \int_{-1}^1 t(1 - 2t + t^2) dt = \int_{-1}^1 (t - 2t^2 + t^3) dt = \left[\frac{1}{2}t^2 - \frac{2}{3}t^3 + \frac{1}{4}t^4\right]_{-1}^1 = \left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) - \left(\frac{1}{2} + \frac{2}{3} + \frac{1}{4}\right) = -\frac{4}{3}$
35. $\int_1^4 \left(\frac{4 + 6u}{\sqrt{u}}\right) du = \int_1^4 \left(\frac{4}{\sqrt{u}} + \frac{6u}{\sqrt{u}}\right) du = \int_1^4 (4u^{-1/2} + 6u^{1/2}) du = \left[8u^{1/2} + 4u^{3/2}\right]_1^4 = (16 + 32) - (8 + 4) = 36$
36. $\int_0^1 \frac{4}{1 + p^2} dp = \left[4 \arctan p\right]_0^1 = 4 \arctan 1 - 4 \arctan 0 = 4\left(\frac{\pi}{4}\right) - 4(0) = \pi$
37. $\int_{\pi/6}^{\pi/3} (4 \sec^2 y) dy = \left[4 \tan y\right]_{\pi/6}^{\pi/3} = 4 \tan \frac{\pi}{3} - 4 \tan \frac{\pi}{6} = 4 \cdot \sqrt{3} - 4 \cdot \frac{1}{\sqrt{3}} = 4\sqrt{3} - \frac{4\sqrt{3}}{3} = \frac{8\sqrt{3}}{3}$ or $\frac{8}{\sqrt{3}}$
38. $\int_0^{\pi/2} (\sqrt{t} - 3 \cos t) dt = \int_0^{\pi/2} (t^{1/2} - 3 \cos t) dt = \left[\frac{2}{3}t^{3/2} - 3 \sin t\right]_0^{\pi/2} = \left[\frac{2}{3}\left(\frac{\pi}{2}\right)^{3/2} - 3 \sin \frac{\pi}{2}\right] - 0 = \frac{2}{3}\left(\frac{\pi}{2}\right)^{3/2} - 3 \cdot 1 = \frac{2}{3}\left(\frac{\pi}{2}\right)^{3/2} - 3$
39. $\int_0^1 x(\sqrt[3]{x} + \sqrt[4]{x}) dx = \int_0^1 (x^{4/3} + x^{5/4}) dx = \left[\frac{3}{7}x^{7/3} + \frac{4}{9}x^{9/4}\right]_0^1 = \left(\frac{3}{7} + \frac{4}{9}\right) - 0 = \frac{55}{63}$
40. $\int_1^4 \frac{\sqrt{y} - y}{y^2} dy = \int_1^4 \left(\frac{\sqrt{y}}{y^2} - \frac{y}{y^2}\right) dy = \int_1^4 (y^{-3/2} - y^{-1}) dy = \left[-2y^{-1/2} - \ln|y|\right]_1^4 = \left[-\frac{2}{\sqrt{y}} - \ln|y|\right]_1^4 = (-1 - \ln 4) - (-2 - \ln 1) = 1 - \ln 4$
41. $\int_1^2 \left(\frac{x}{2} - \frac{2}{x}\right) dx = \left[\frac{1}{4}x^2 - 2 \ln|x|\right]_1^2 = (1 - 2 \ln 2) - \left(\frac{1}{4} - 2 \ln 1\right) = \frac{3}{4} - 2 \ln 2$
42. $\int_0^1 (5x - 5^x) dx = \left[\frac{5}{2}x^2 - \frac{5^x}{\ln 5}\right]_0^1 = \left(\frac{5}{2} - \frac{5}{\ln 5}\right) - \left(0 - \frac{1}{\ln 5}\right) = \frac{5}{2} - \frac{4}{\ln 5}$