

Complete the Square Examples

$$\textcircled{1} \int \frac{dx}{x^2 + 8x + 25} = \int \frac{dx}{(x^2 + 8x + 16) + 9}$$

$$= \int \frac{dx}{(x+4)^2 + 9}$$

$$\boxed{\begin{array}{l} u = x+4 \\ du = dx \end{array}}$$

$$= \int \frac{1}{u^2 + 9} du$$

$$= \frac{1}{3} \arctan\left(\frac{u}{3}\right) + C$$

$$= \frac{1}{3} \arctan\left(\frac{x+4}{3}\right) + C$$

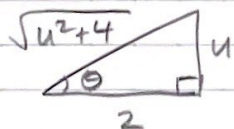
$$\textcircled{2} \int \frac{1}{\sqrt{x^2 + 2x + 5}} dx = \int \frac{1}{\sqrt{(x^2 + 2x + 1) + 4}} dx$$

$$= \int \frac{1}{\sqrt{(x+1)^2 + 4}} dx$$

$$\boxed{\begin{array}{l} u = x+1 \\ du = dx \end{array}}$$

$$= \int \frac{1}{\sqrt{u^2 + 4}} du$$

$$\left. \begin{array}{l} \text{Let } u = 2 \tan \theta \\ du = 2 \sec^2 \theta d\theta \end{array} \right\}$$



from triangle,

$$\frac{\sqrt{u^2 + 4}}{2} = \sec \theta$$

$$\sqrt{u^2 + 4} = 2 \sec \theta$$

$$= \int \frac{1}{2 \sec \theta} \cdot 2 \sec^2 \theta d\theta$$

$$= \int \sec \theta d\theta$$

$$= \ln |\sec \theta + \tan \theta| + C$$

$$= \ln \left| \frac{\sqrt{u^2 + 4}}{2} + \frac{u}{2} \right| + C$$

$$= \ln \left| \frac{\sqrt{(x+1)^2 + 4}}{2} + \frac{x+1}{2} \right| + C$$