

<u>Maclaurin Series</u>			<u>Radius of Convergence</u>
$\frac{1}{1-x}$	$= \sum_{n=0}^{\infty} x^n$	$= 1 + x + x^2 + x^3 + \dots$	$R = 1$
e^x	$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$	$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$	$R = \infty$
$\sin x$	$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$	$R = \infty$
$\cos x$	$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$	$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$	$R = \infty$
$\arctan(x)$	$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$	$= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	$R = 1$
$\ln(1+x)$	$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$	$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$	$R = 1$

1. Using the table above, match the following functions with their corresponding Maclaurin series.

(a) $f(x) = e^{x^2/2}$

I. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

II. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{4^n (2n)!}$

(b) $g(x) = \cos\left(\frac{x}{2}\right)$

III. $\sum_{n=1}^{\infty} \frac{-x^n}{n}$

(c) $h(x) = \ln(1-x)$

IV. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{2n+1}$

V. $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!}$

(d) $k(x) = x^3 \arctan x$

VI. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+3}}{2n+1}$

2. Use the table of known Maclaurin series to find the sum of each series.

Hint: **Do the left column first.** Then use your answer from the left column to find the sum of the corresponding series in the right column. Part (h) is a challenge problem :)

(a) $\sum_{n=0}^{\infty} (-1)^n \frac{(1/2)^{2n+1}}{(2n+1)}$

(e) $\sum_{n=1}^{\infty} (-1)^n \frac{(1/2)^{2n+1}}{(2n+1)}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n3^n}$

(f) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n3^n}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{3^{2n+1} (2n+1)!}$

(g) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{3^{2n} (2n+1)!}$

(d) $\sum_{n=0}^{\infty} \frac{1}{n!}$

(h) $\sum_{n=0}^{\infty} \frac{1}{(2n)!}$