

I) CANONICAL ENSEMBLE

LEC. 9

①

SP915

A) 1st DERIVATION OF $P(\sigma_r) = \frac{e^{-\beta E_r}}{\sum_j e^{-\beta E_j}}$ (PATHRIA 3.1)

1) $\Omega^0 = \Omega'(E') \Omega(E_r) = \left(\begin{matrix} \text{NO. of states in} \\ \text{Large reservoir} \end{matrix} \right) \cdot \left(\begin{matrix} \text{NO. of states in} \\ \text{system} \end{matrix} \right)$

2) $E^0 = E' + E_r \Rightarrow$ (Everything insulated)

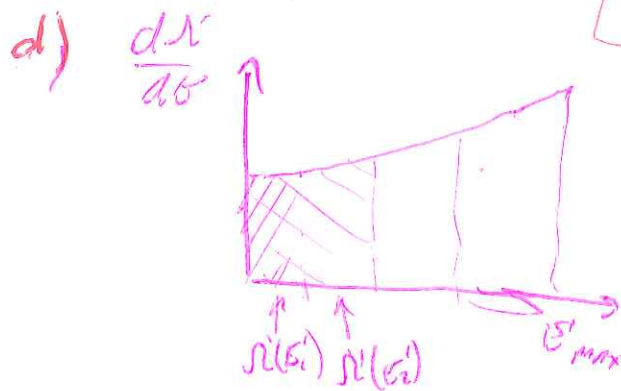
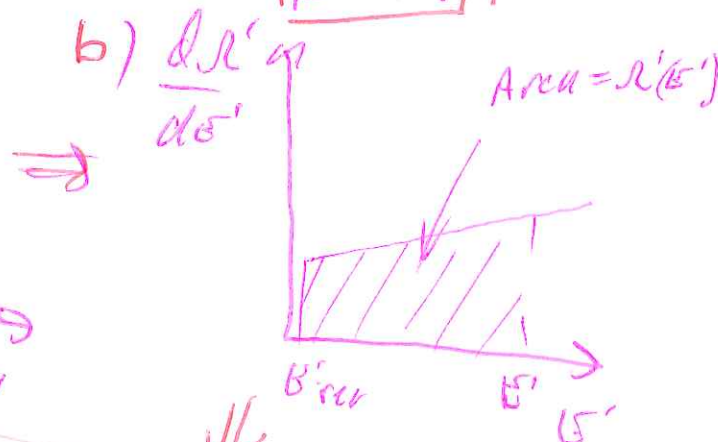
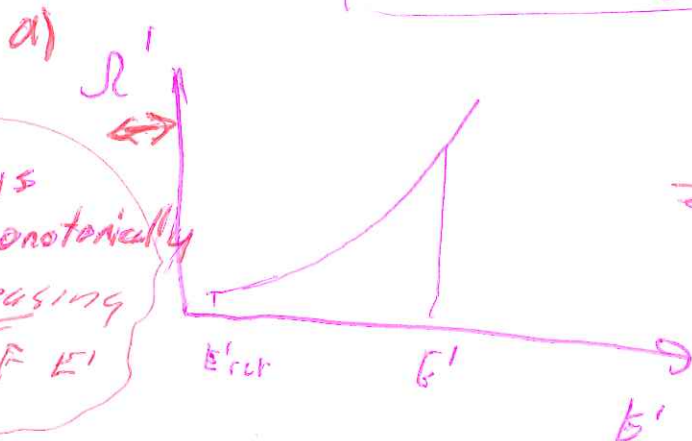
3) Once E' fixed, E_r known?

$\Rightarrow P(E') = P(E_r) \Leftrightarrow$



NEXT:
4) IT'S CLAIMED THAT:

$P(\sigma') \propto \Omega'(E')$ weak proof:



$\Omega'(E') = \int \left(\frac{d\Omega'}{dE'} \right) dE'$

e) prob. of reservoir being in E' $= \frac{\Omega(E')}{\Omega_{tot}} = P(E')$

f) $\Rightarrow P(\sigma') \propto \Omega'(E')$
claim proved

5) so far, we have:

i) $P(E') \equiv P(E_r)$

ii) $P(E') \propto \Omega'(E')$

NEXT STOP (PARTIAL);

$$\ln \Omega'(E') = \ln(\Omega^0 - E_r)$$

$$= \ln \Omega^0 + \frac{\partial \ln \Omega'}{\partial E'} \bigg|_{E_0} (-E_r)$$

$$\equiv \beta'$$

(2.1) applies
in and
out
at
equilibrium

$\therefore$

$$\ln \Omega'(E') = C_0 - \beta' E_r \quad \text{(from (ch. 1))} \quad (2.1)$$

6) now, FOCUS ON EQUILIBRIUM

at equilibrium, we know that (ch. 1):

$$\boxed{\beta = \beta'}$$

(2.2)

$$\left(\frac{\partial \ln \Omega}{\partial E} \bigg|_{N, V} = \frac{\partial \ln \Omega'}{\partial E'} \bigg|_{N', V'} \right)$$

6b) i. using (2.2) in (2.1) at equilibrium:

$$\Omega'(E') = A_0 e^{-\beta E_r}$$

or since $\Omega'(E') \propto P(E_r)$

$$\text{then } \boxed{P(E_r) = B_0 e^{-\beta E_r}}$$

6) b) GET B_0 : $\sum_{r=1}^{N \text{ of states}} P(E_r) = 1 = B_0 \sum_{r=1} e^{-\beta E_r}$

$$P(E_r) = \frac{e^{-\beta E_r}}{\sum_i e^{-\beta E_i}}$$

③

ASIDE: DYNAMICS / ENERGY STATES
HAMILTONIAN =
 OF CLASSICAL NON-DISSIPATIVE
SYSTEMS - N-BODY SYSTEMS
 N large

Recipe

1) Define / guess A SYSTEM

HAMILTONIAN, H

NOTES:
 IT ONLY
CONSERVATIVE
 forces
 allowed.

$F_i = -\frac{\partial V}{\partial q_i}$
 in Hamiltonian
 models

$$H = \underbrace{\sum_{i=1}^{3N} \frac{p_i^2}{2m_i}}_{\substack{\text{total} \\ \text{K.E.} \\ \text{or} \\ N \\ \text{ptcls}}} + \underbrace{V(q_1, q_2, \dots, q_{3N})}_{\substack{\text{total} \\ \text{potential} \\ \text{energy} \\ \text{of } N \text{ ptcls}}} = E$$

total
conserved
sys
energy

2) MAKE USE OF NEWTON'S
 LAW IN GUSS OF HAMILTON'S
 EQNS.

$$(1.1) \left\{ \begin{array}{l} \ddot{q}_i = \frac{\partial H}{\partial p_i} \\ \dot{p}_i = -\frac{\partial H}{\partial q_i} \end{array} \right. \quad i=1, 2, 3, \dots, 3N$$

(Newton's 2nd law)

$\{q_i = \text{instantaneous position with gen dir}$
 $(p_i = \text{instantaneous momentum with generalized direction})$

3) For N-Body classical problems, with N large, the phase space picture approach turns out to be super-useful (Pathria Ch. 2)

A [Aside: Phase space for N-ptcl system:]

a) THE INSTANTANEOUS dynamics of our N ptcls are pictured w/in a $6N$ -dim phase space.

b) Each of the $6N$ -coords in this space correspond to the $3N$ momentum components plus the $3N$ positions coordinates for the N ptcls

phase space con't

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(3)

c) To study the statistical behavior of our N -ptcl system, imagine

$\boxed{\tilde{N}}$ replica systems $\equiv$

An ensemble

d) Collect the dynamical

behavior/evolution of all $\tilde{N}$ replicants

in a single $6N-D$

phase space.

e) Pictorially, we have/see a swarm of $\tilde{N}$

pts, evolving/moving through our $6N-D$ phase space.

(where each pt n ^(say the n^{th} point) represents the instantaneous dynamical

phase space config

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5/15/15 (4)

state of representative sys.
 α w/in the ensemble.

f) The dynamical evolution
of all N state points
(in the $6N$ -Dim phase
space) is governed by
the Hamiltonian eqns
(1.1).

g) THE DENSITY, $\rho = \rho(\underline{p}, \underline{q}, t)$, of
state points w/in
the phase space gives
the instantaneous

probability of ~~the~~ observing
our actual system
at the given state point.

$$(\underline{p}, \underline{q}) = (p_1, p_2, \dots, p_{3N}; q_1, q_2, \dots, q_{3N})$$

phase space coin 5/9 is loc 9 (5)
 h) in terms of $P = P(\underline{q}, \underline{p}, t)$,
 the ensemble average
 of any given phase-
 space-dependent ($\equiv$ dynamically
 determined) function
 f is given by

$$\langle f \rangle = \int_{\Omega} f(\underline{q}, \underline{p}) P(\underline{q}, \underline{p}, t) d\underline{q} d\underline{p}$$

$\underbrace{\quad}_{3N \text{ components}}$

$$\int_{\Omega} P(\underline{q}, \underline{p}) d\underline{q} d\underline{p}$$

$\Omega =$ "volume" of our
 system's phase
 space

END ASIDE phase space

4) THE MOST IMPORTANT
 RESULT FOR OUR
 PURPOSES IS THAT FOR
stationary ensembles
 ($\equiv \rho \neq f_n(t)$),

$$\boxed{P(\underline{q}, \underline{p}) \propto \exp[-H(\underline{q}, \underline{p})\beta]}$$

(See the rest of the
 somewhat (if you've had exposure to
straight forward ^{Hamiltonian} ^{quantum} argument
 in 2.2 path integral)