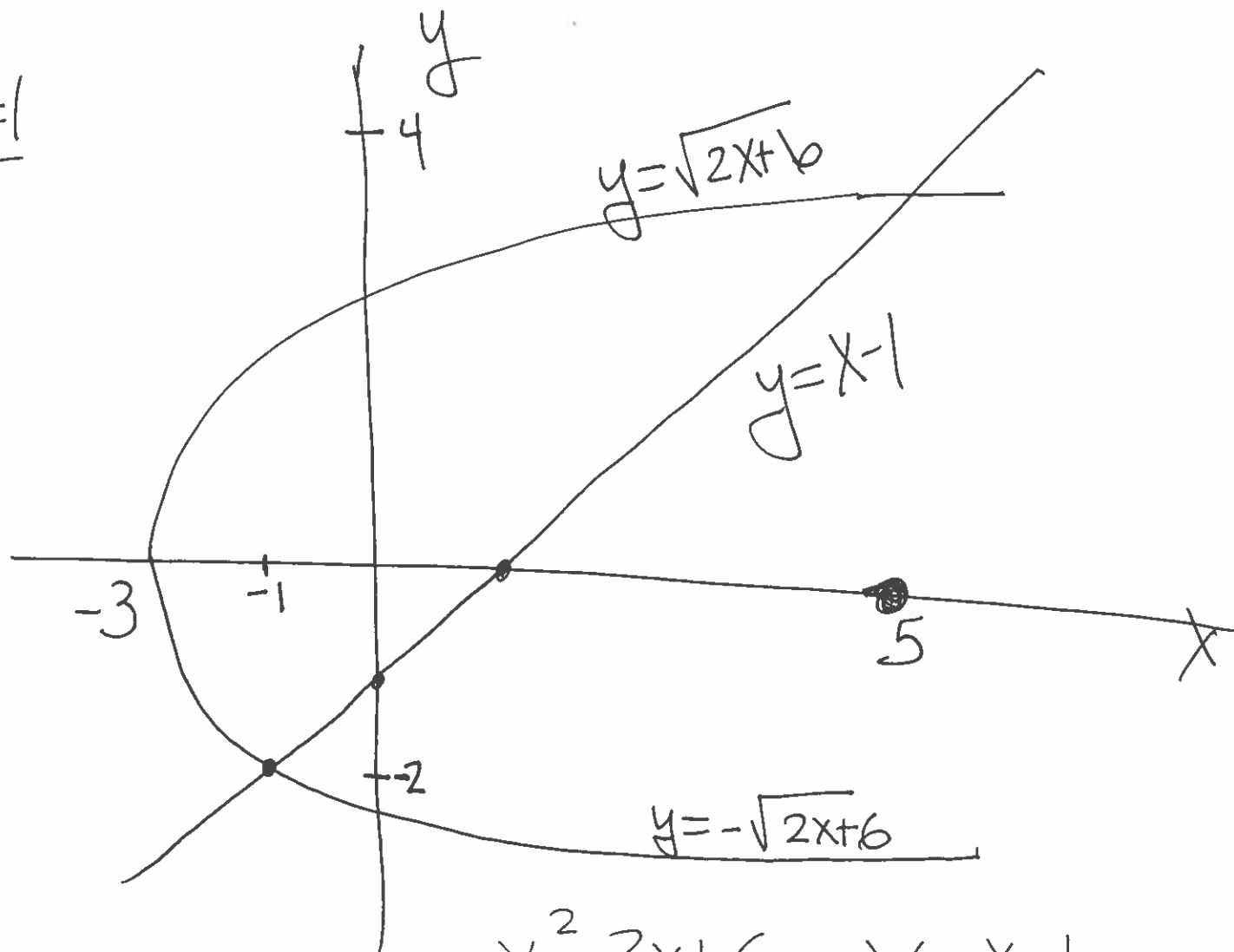


PRACTICE exam 2A Solutions

#1



$$\begin{aligned}(x-1)^2 &= 2x+6 \\ x^2 - 2x + 1 &= 2x + 6\end{aligned}$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$$x = -1, 5$$

$$y^2 = 2x + 6 \quad y = x - 1$$

$$x = -1 \quad y = -2$$

$$x = 5 \quad y = 4$$

$$y = \pm \sqrt{2x+6}$$

using variable x

$$\text{Area} = \int_{-3}^{-1} (\sqrt{2x+6} - (-\sqrt{2x+6})) dx + \int_{-1}^5 (\sqrt{2x+6} - (x-1)) dx =$$
$$2 \int_{-3}^{-1} \sqrt{2x+6} dx + \int_{-1}^5 (\sqrt{2x+6} - x + 1) dx$$

#2 $x^3 - 3x^2 - 3x - 4 = (x-4)(\underbrace{x^2 + x + 1}_{\text{irreducible}})$

$$\frac{x^2 + 4x + 2}{x^3 - 3x^2 - 3x - 4} = \frac{x^2 + 4x + 2}{(x-4)(x^2 + x + 1)} =$$

$$\frac{A}{x-4} + \frac{Bx + C}{x^2 + x + 1}$$

#3 $\int \frac{1}{x^2 \sqrt{x^2 - 2}} dx$

$$x = \sqrt{2} \sec \theta$$

$$dx = \sqrt{2} \sec \theta \tan \theta d\theta$$

$$x^2 - 2 = 2 \sec^2 \theta - 2 =$$

$$2(\sec^2 \theta - 1) = 2 \tan^2 \theta$$

$$\int \frac{\sqrt{2} \sec \theta \tan \theta}{2 \sec^2 \theta \sqrt{2 \tan^2 \theta}} d\theta = \int \frac{\sqrt{2} \sec \theta \tan \theta}{2 \sec^2 \theta \sqrt{2} \tan \theta} d\theta = \frac{1}{2} \int \frac{1}{\sec \theta} d\theta$$

$$\#4 \quad 2x^2 - 2x + 5 = A(x^2 + 4) + (Bx + C)(x - 1)$$

$$\text{let } x=1 \quad 2(1)^2 - 2(1) + 5 = A(1^2 + 4) + (B(1) + C)(1 - 1)$$

$$5 = 5A \quad \boxed{A=1}$$

$$\#5 \quad \int x^2 \sqrt{x^2 + 1} \, dx$$

$$x = \tan \theta$$

$$dx = \sec^2 \theta \, d\theta$$

$$x^2 + 1 = \tan^2 \theta + 1 = \sec^2 \theta$$

$$\int \tan^2 \theta \sqrt{\tan^2 \theta + 1} \sec^2 \theta \, d\theta = \int \tan^2 \theta \sqrt{\sec^2 \theta} \sec^2 \theta \, d\theta$$

$$\int \tan^2 \theta \sec^3 \theta \, d\theta = \int (\sec^2 \theta - 1) \sec^3 \theta \, d\theta =$$

$$\int (\sec^5 \theta - \sec^3 \theta) \, d\theta = \int \sec^5 \theta \, d\theta - \int \sec^3 \theta \, d\theta$$

$$\int \sec^3 \theta \, d\theta = \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta|$$

$$\int \sec^5 \theta \, d\theta = \int \sec^3 \theta \sec^2 \theta \, d\theta \quad \begin{array}{l} u = \sec^3 \theta \quad dv = \sec^2 \theta \, d\theta \\ du = 3\sec^2 \theta \sec \theta \tan \theta \, d\theta \end{array}$$

$$= \sec^3 \theta \tan \theta - 3 \int \sec^3 \theta \tan^2 \theta \, d\theta = \quad \quad \quad V = \tan \theta$$

$$\sec^3 \theta \tan \theta - 3 \int \sec^3 \theta (\sec^2 \theta - 1) d\theta =$$

$$\sec^3 \theta \tan \theta - 3 \int \sec^5 \theta d\theta + 3 \int \sec^3 \theta d\theta$$

$$\int \sec^5 \theta d\theta = \sec^3 \theta \tan \theta - 3 \int \sec^3 \theta d\theta + 3 \int \sec^3 \theta d\theta$$

$$4 \int \sec^5 \theta d\theta = \sec^3 \theta \tan \theta + 3 \int \sec^3 \theta d\theta$$

$$\int \sec^5 \theta d\theta = \frac{1}{4} \sec^3 \theta \tan \theta + \frac{3}{4} \int \sec^3 \theta d\theta$$

$$\int \tan^2 \theta \sec^3 \theta d\theta = \int \sec^5 \theta d\theta - \int \sec^3 \theta d\theta$$

$$\int \tan^2 \theta \sec^3 \theta d\theta = \frac{1}{4} \sec^3 \theta \tan \theta + \frac{3}{4} \int \sec^3 \theta d\theta - \int \sec^3 \theta d\theta$$

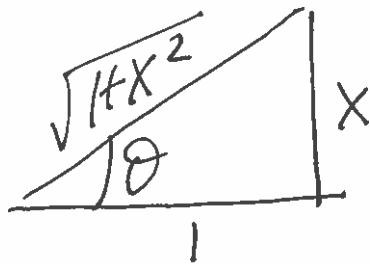
$$\int \tan^2 \theta \sec^3 \theta d\theta = \frac{1}{4} \sec^3 \theta \tan \theta - \frac{1}{4} \int \sec^3 \theta d\theta$$

$$\int \tan^2 \theta \sec^3 \theta d\theta = \frac{1}{4} \sec^3 \theta \tan \theta - \frac{1}{4} \left[\frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| \right]$$

$$\int \tan^2 \theta \sec^3 \theta d\theta = \frac{1}{4} \sec^3 \theta \tan \theta - \frac{1}{8} \sec \theta \tan \theta - \frac{1}{8} \ln |\sec \theta + \tan \theta|$$

$$x = \tan \theta$$

$$\sec \theta = \sqrt{1+x^2}$$



$$\int x^2 \sqrt{x^2+1} dx = \frac{1}{4} (x^2+1)^{3/2} \cdot x - \frac{1}{8} x \sqrt{x^2+1} - \frac{1}{8} \ln |\sqrt{1+x^2} + x| + C$$

#6
$$\frac{7x^2 + 4x + 27}{(x-2)(x^2+5)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+5}$$

$$7x^2 + 4x + 27 = A(x^2+5) + (Bx+C)(x-2)$$

Plug in $x=2$ $28 + 8 + 27 = 9A$ $63 = 9A$ $A=7$

$$7x^2 + 4x + 27 = 7x^2 + 35 + Bx^2 - 2Bx + Cx - 2C$$

$$7x^2 + 4x + 27 = (7+B)x^2 + (C-2B)x + (35-2C)$$

$$7 = 7+B \quad B=0 \quad 4 = C-2B \quad B=0 \quad C=4$$

CHECK: $27 = 35 - 2 \cdot 4$ $27 = 27$ ✓

$$\int \frac{7x^2 + 4x + 27}{(x-2)(x^2+5)} dx = 7 \int \frac{1}{x-2} dx + 4 \int \frac{1}{x^2+5} dx$$

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \operatorname{Arctan}\left(\frac{x}{a}\right)$$

$$\text{Answer: } 7 \ln|x-2| + \frac{4}{\sqrt{5}} \operatorname{Arctan}\left(\frac{x}{\sqrt{5}}\right) + C$$

$$\#7 \quad \int_0^1 \frac{1}{\sqrt{1-x}} dx = \lim_{t \rightarrow 1^-} \int_0^t \frac{1}{\sqrt{1-x}} dx$$

$$\int \frac{1}{\sqrt{1-x}} dx \quad u=1-x \quad \boxed{-du=dx}$$

$$du = -dx$$

$$\int \frac{1}{\sqrt{u}} (-du) = - \int u^{-1/2} du = - \frac{u^{1/2}}{1/2} =$$

$$-2\sqrt{u} = -2\sqrt{1-x}$$

$$\int_0^1 \frac{1}{\sqrt{1-x}} dx = \lim_{t \rightarrow 1^-} (-2\sqrt{1-x}) \Big|_0^t = \lim_{t \rightarrow 1^-} [-2\sqrt{1-t} + 2] = 2$$

#8 $x^2 + \ln x \geq x^2$ for $x \geq e$

$$0 \leq \frac{1}{x^2 + \ln x} \leq \frac{1}{x^2} \text{ for } x \geq e$$

$$0 \leq \int_e^{\infty} \frac{1}{x^2 + \ln x} dx \leq \int_e^{\infty} \frac{1}{x^2} dx$$

converges
p-integral
with $p=2>1$

Also converges
by Direct Comparison Test.