Optimal Bidding Strategies in Network-Constrained Demand Response: A Distributed Aggregative Game Theoretic Approach

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European Control Conference, Stockholm, Sweden

June 25-28, 2024

Overview

1 Introduction

2 Problem description and game formulation

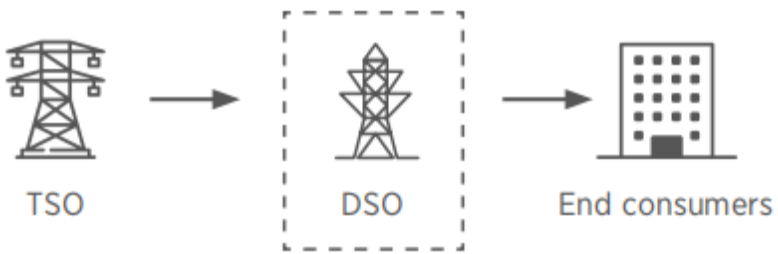
3 Algorithm design

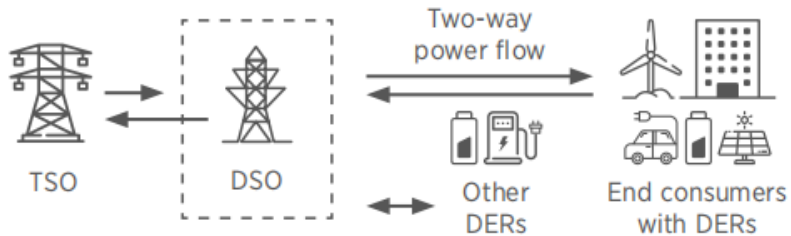
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Introduction

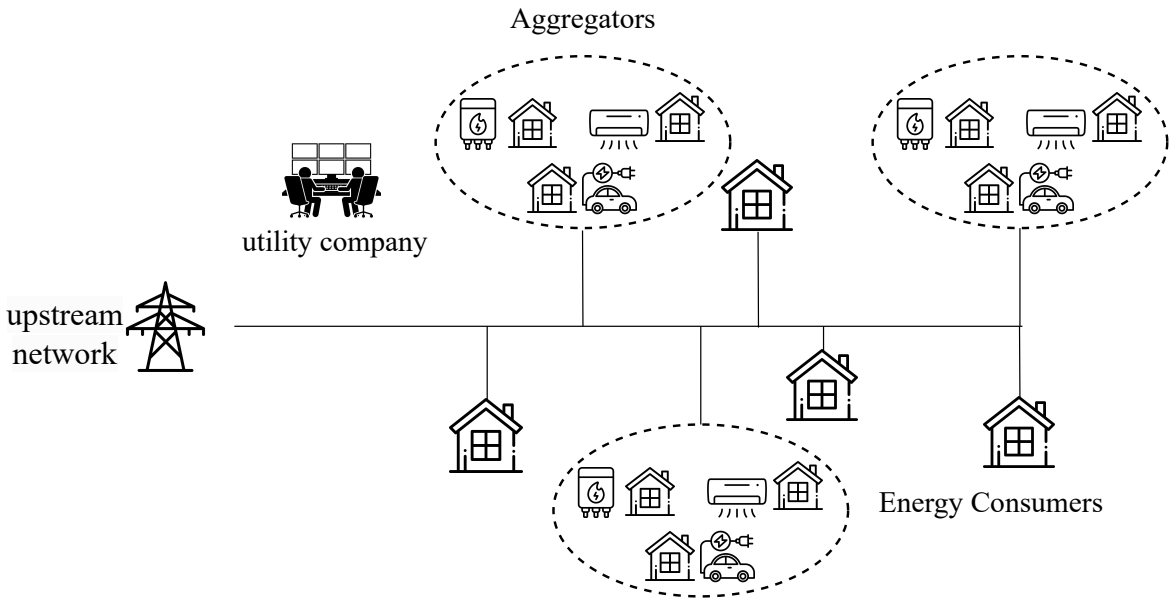
- Increasing penetration of uncertain and volatile renewable energy generation into power system
- Increasing need for flexibility (adjustment of energy generation or consumption)
- Aggregators (energy consumers) providing flexibility for balancing with DERs via demand response





Centralized Power Plants $\longrightarrow$ Distributed Energy Resources (DERs)

Problem description



$$\sum_{n \in \mathcal{N}} x_n = r$$

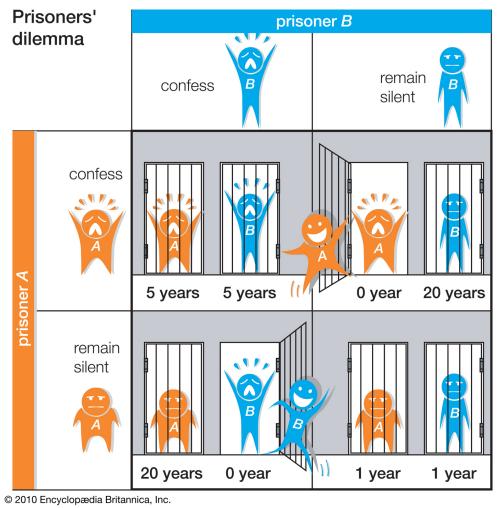
- **Utility Company (UC)**: procures flexibility services for energy balancing in distribution networks
- **Aggregators**: provide flexibility by adjusting their DERs

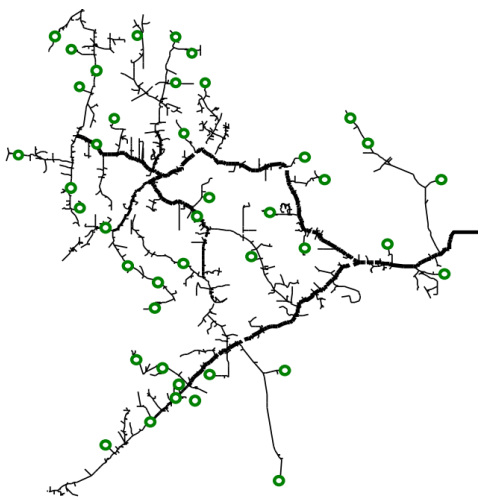
Problem description



Balancing/Flexibility market

- Self-interested behaviors of aggregators
- Distribution network operation constraints
- Limited information sharing



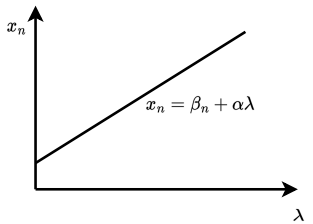




Market operation

- Utility company: flexibility requirement r
- Each aggregator: linear supply function bidding

$$x_n = \beta_n + \alpha\lambda$$



- Utility company:

$$\text{Clearing Price: } \lambda = (r - \mathbb{1}^\top \beta)/(\alpha N)$$
$$\text{Allocated flexibility: } x_n = (r - \mathbb{1}^\top \beta)/N + \beta_n$$

- Utility company: physical network constraint

$$-\hat{f}_l \leq \sum_{n=1}^{N} \pi_{ln}(e_n - x_n) \leq \hat{f}_l, \forall\, l \in \mathcal{L}$$

Game formulation

Economic interest:

$$\min\ J_n(x_n, \lambda) = \underbrace{C_n(x_n)}_{\text{payment to consumers}} - \underbrace{\lambda x_n}_{\text{payment from UC}}$$

$$\text{s.t.} \quad \underbrace{0 \le x_n \le \hat{x}_n}_{\text{local flexibility constraint}}$$

where $C_n(x_n) = q_n(x_n)x_n$ with pricing function $q_n(\cdot) := a_n x_n + b_n$.

Objective:

$$\min_{\beta_n}\ \bar{J}_n(\beta_n, \beta_{-n}) = \underbrace{C_n\big((r - \mathbf{1}^\top\beta)/N + \beta_n\big)}_{C_n(x_n)} - \underbrace{(r - \mathbf{1}^\top\beta + N\beta_n)(r - \mathbf{1}^\top\beta)/(\alpha N^2)}_{\lambda x_n}$$

Coupling constraint:

$$\tilde{A}\beta \le d$$

Game formulation

Game among aggregators:

$$\mathcal{G} = \{\mathcal{N}, K, \text{col}(\bar{J}_n(\beta_n, \beta_{-n}))_{n\in\mathcal{N}}\},$$

where $K = \{\beta \in \Omega \mid \tilde{A}\beta \leq d\}$.

Definition

A point $\beta^* \in K$ is a generalized Nash equilibrium (GNE) of the game, if for all $n \in \mathcal{N}$, the following holds,

$$\bar{J}_n(\beta_n, \beta^*_{-n}) \geq \bar{J}_n(\beta^*_n, \beta^*_{-n}), \quad \forall\ \beta_n \in K_n(\beta^*_{-n}).$$

Game formulation

Lemma

The game $\mathcal{G}$ has a v-GNE, given by the solution $\beta^* \in K$ of the variational inequality $\mathrm{VI}(K, F)$, i.e.

$$(\beta - \beta^*)^\top F(\beta^*) \geq 0, \quad \forall \beta \in K.$$

Here F is the pseudo-gradient mapping of the game $\mathcal{G}$,

$$F := \mathrm{col}(f_n(\beta_n, \beta_{-n}))_{n \in \mathcal{N}},$$

where

$$\begin{aligned} f_n(\beta_n, \beta_{-n}) &:= \frac{\partial}{\partial \beta_n} \bar{J}_n(\beta_n, \beta_{-n}) \\ &= \frac{N-1}{N} C_n'(x_n) + \frac{(\mathbb{1}^\top \beta - r)(N-2) + N\beta_n}{\alpha N^2}, \end{aligned}$$

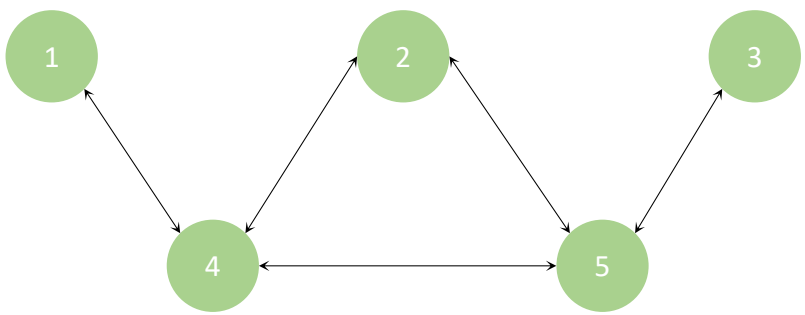
Algorithm design

Gradient-based algorithm:

$$\beta_n^{k+1} = \beta_n^k - \tau f_n(\beta_n^k, \beta_{-n}^k)$$

Full Information setting: knowing all β_{-n} and sharing β_n with all aggregators.

Challenge: What if each aggregator does not know all β_{-n} or is not willing to share β_n with neighbouring aggregators?



Algorithm design

- σ_n: local estimate of the aggregative bid $\mathbb{1}^\top \beta / N$;
- λ_n: local estimates of the multipliers of coupling constraint;

$$\hat{f}_n(\beta_n, \sigma_n) := \frac{N-1}{N} C_n' \left(\frac{r - N\sigma_n}{N} + \beta_n \right) + \frac{(N\sigma_n - r)(N-2) + N\beta_n}{\alpha N^2},$$

- ψ_n, z_n: two auxiliary variables with the aim of reaching the consensus on the local estimates σ_n's and λ_n's.

Algorithm design

Initialization: For each $n \in \mathcal{N}$, set $\beta_n^0 \in \Omega_n$, $\sigma_n^0 \in \mathbb{R}$, $\psi_n^0 \in \mathbb{R}$, $z_n \in \mathbb{R}^M$, $\lambda_n \in \mathbb{R}_+^M$.

Iterate until convergence:

Local variable update at the kth step:

$$\beta_n^{k+1} = \text{proj}_{\Omega_n}[\beta_n^k - \tau_n(\hat{f}_n(\beta_n^k, \sigma_n^k) + \tilde{A}_n^\top \lambda_n^k)]$$

$$\psi_n^{k+1} = \psi_n^k + \upsilon_n \sum_{m \in \mathcal{N}_n} w_{nm}(\sigma_n^k - \sigma_m^k)$$

$$\sigma_n^{k+1} = \sigma_n^k + \rho_n\big(\kappa(\beta_n^k - \sigma_n^k) - \sum_{m \in \mathcal{N}_n} w_{mn}(2(\psi_n^{k+1} - \psi_m^{k+1}) - (\psi_n^k - \psi_m^k))\big)$$

$$z_n^{k+1} = z_n^k + \delta_n \sum_{m \in \mathcal{N}_n} w_{nm}(\lambda_n^k - \lambda_m^k)$$

$$\lambda_n^{k+1} = \text{proj}_{\mathbb{R}_+^M}[\lambda_n^k - \eta_n\big(\sum_{m \in \mathcal{N}_n} w_{nm}(\lambda_n^k - \lambda_m^k) + d_n + \tilde{A}_n(\beta_n^k - 2\beta_n^{k+1}) + \sum w_{nm}(2(z_n^{k+1} - z_m^{k+1}) - (z_n^k - z_m^k))\big)]$$

Algorithm design

The algorithm in a compact form:

$$
\begin{aligned}
\beta^{k+1} &= \mathrm{proj}_{\Omega}[\beta^k - \tau(\hat{F}(\beta^k, \sigma^k) + \bar{A}^\top \lambda^k)] \\
\psi^{k+1} &= \psi^k + \upsilon L \sigma^k \\
\sigma^{k+1} &= \sigma^k + \rho(\kappa(\beta^k - \sigma^k) - L(2\psi^{k+1} - \psi^k)) \\
z^{k+1} &= z^k + \delta L \lambda^k \\
\lambda^{k+1} &= \mathrm{proj}_{\mathbb{R}^{NM}_+}[\lambda^k - \eta(L\lambda^k + \bar{d} - \bar{A}(2\beta^{k+1} - \beta^k) \\
&\quad + L(2z^{k+1} - z^k))]
\end{aligned}
$$

where $\hat{F}(\beta, \sigma) := \mathrm{col}(\hat{f}_n(\beta_n, \sigma_n))_{n \in \mathcal{N}}$ and L denoting the Laplacian matrix of G.

Question: Could this algorithm converge to the v-GNE of our game?

Algorithm design

Theorem 1

The algorithm is equivalent to the forward-backward iteration

$$\omega^{k+1} = \mathcal{V}_\Phi \circ \mathcal{U}_\Phi(\omega^k),$$

in particular, its fix point coincides with the steady state $\omega^* = (\beta^*, \psi^*, \sigma^*, z^*, \lambda^*)$ of the algorithm. Moreover, β^* is a v-GNE of the game $\mathcal{G}$.

$\omega := \mathrm{col}(\beta, \psi, \sigma, z, \lambda)$, $\mathcal{V}_\Phi := (\mathrm{Id} + \Phi^{-1}\mathcal{B})^{-1}$, $\mathcal{U}_\Phi := (\mathrm{Id} - \Phi^{-1}\mathcal{A})$, and

$$\mathcal{A} := \begin{bmatrix} \hat{F}(\beta, \sigma) \\ 0 \\ \kappa(\sigma - \beta) \\ 0 \\ \bar{d} + L_\lambda \lambda \end{bmatrix}, \mathcal{B} := \begin{bmatrix} \mathrm{N}_\Omega(\beta) + \bar{A}^\top \lambda \\ -L_\sigma \sigma \\ L_\sigma \psi \\ -L_\lambda \lambda \\ \mathrm{N}_{\mathbb{R}^M_+}(\lambda) - \bar{A}x + L_\lambda z \end{bmatrix}, \Phi := \begin{bmatrix} \tau^{-1} & 0 & 0 & 0 & -\bar{A}^\top \\ 0 & \upsilon^{-1} & L_\sigma & 0 & 0 \\ 0 & L_\sigma & \rho^{-1} & 0 & 0 \\ 0 & 0 & 0 & \delta^{-1} & L_\lambda \\ -\bar{A} & 0 & 0 & L_\lambda & \eta^{-1} \end{bmatrix}.$$

Algorithm design

Theorem 2

If the designed parameters are chosen properly, any sequence generated by the algorithm converges to the steady state$\omega^* = (\beta^*, \psi^*, \sigma^*, z^*, \lambda^*)$.

$$\kappa \in \left(\sqrt{\max_{n\in\mathcal{N}} \mu_n} - \gamma, \sqrt{\min_{n\in\mathcal{N}} \mu_n} + \gamma \right), \tau_n < 2\epsilon, \upsilon_n < 2\epsilon, \delta_n > 2\epsilon,$$

$$\rho_n^{-1} > \lambda_{\max}^2(L) \left(\frac{1}{\max_{n\in\mathcal{N}} \upsilon_n} - \frac{1}{2\epsilon} \right)^{-1} + \frac{1}{2\epsilon},$$

$$\eta_n^{-1} > \|\bar{A}\|^2 \left(\frac{1}{\max_{n\in\mathcal{N}} \tau_n} - \frac{1}{2\epsilon} \right)^{-1} + \lambda_{\max}^2(L) \left(\frac{1}{\max_{n\in\mathcal{N}} \delta_n} - \frac{1}{2\epsilon} \right)^{-1} + \frac{1}{2\epsilon},$$

where $\mu_n := 2a_n \frac{N-1}{N} + \frac{1}{\alpha N}$, $\ell_n := -2a_n \frac{N-1}{N} + \frac{N-2}{\alpha N}$, $\gamma = \sqrt{\frac{N-1}{\alpha N}}$, $\xi = \frac{\epsilon}{\lambda_{\max}(\Phi^{-1})}$, $\epsilon = \min\{\tilde{\epsilon}, 1/\lambda_{\max}(L)\}$, $\tilde{\epsilon} = \frac{\min_{n\in\mathcal{N}} \bar{\epsilon}_n}{\max_{n\in\mathcal{N}} \underline{\epsilon}_n}$,$\bar{\epsilon}_n = -\sqrt{(\mu_n - \kappa)^2 + (\ell_n - \kappa)^2} + \kappa + \mu_n$, and $\underline{\epsilon}_n = \mu_n^2 + \ell_n^2 + 2\kappa^2 + \sqrt{(\mu_n + \ell_n)^2(\mu_n - \ell_n)^2 + 4(\kappa^2 - \mu_n\ell_n)^2}$.

Numerical Study

We consider $N = 5$ aggregators providing flexibility services $r = 600\text{kWh}$, $\alpha = 1$, $\beta_{\text{min}} = 0$, $\beta_{\text{max}} = 150$ and the power flow limits for the four lines are $\hat{f} = [1.40, 6.0, 2.0, 2.0] \times 1000\text{kWh}$.

Table: Simulation data

Aggregator	$a_n(\$/(\text{kWh})^2)$	$b_n(\$/\text{kWh})$	$e_n(\text{kWh})$	$\hat{x}_n(\text{kWh})$
1	0.0050	0.40	1250	250
2	0.0065	0.38	-1300	200
3	0.0085	0.36	1050	250
4	0.0070	0.37	1700	110
5	0.0095	0.80	1480	220

Numerical Study

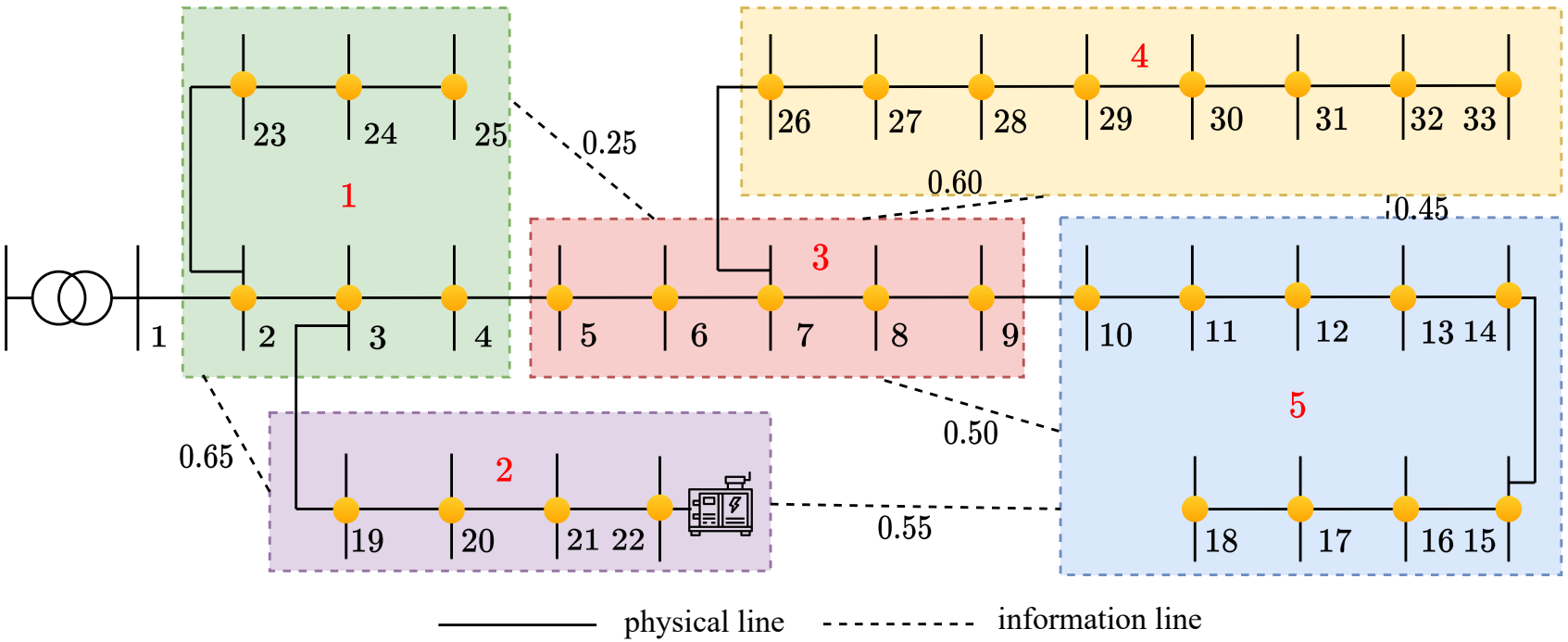


Figure: Physical network and communication network among aggragators

Numerical Study

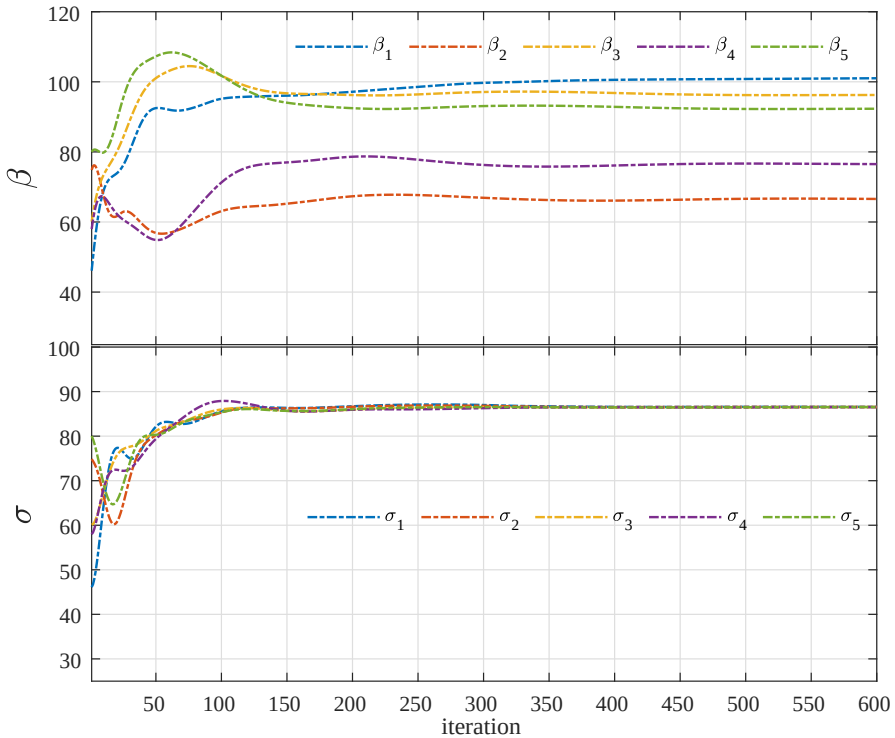


Figure: The evolution of β and σ

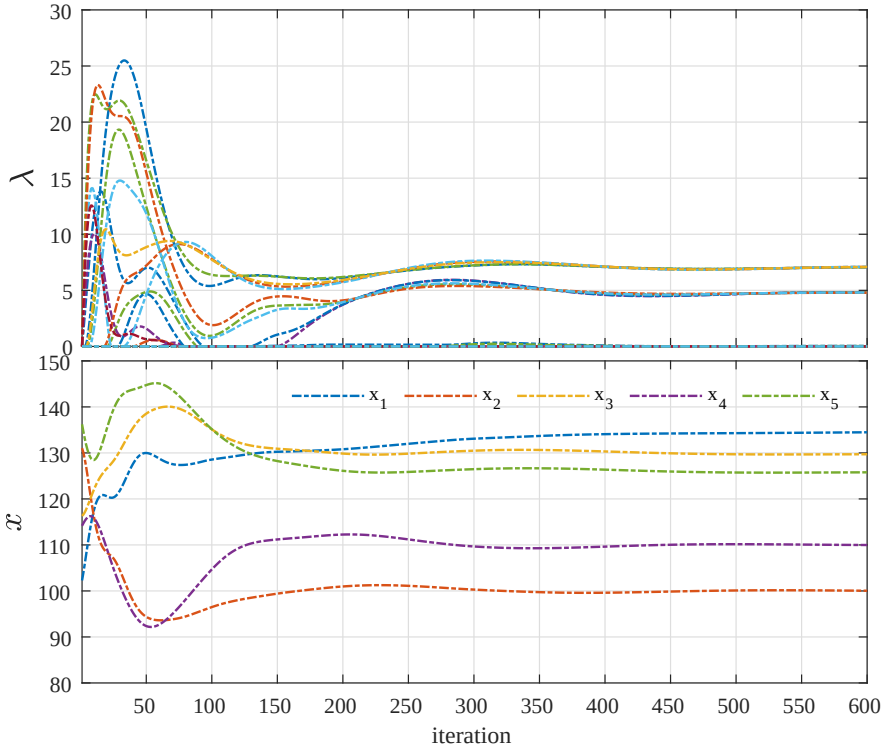


Figure: The evolution of λ and x

Summary

Conclusion

- We proposed a game-theoretic market for procuring flexibility services from aggregators in distribution network.
- A fully distributed algorithm is designed, where only local estimate and auxiliary variables are shared with neighboring aggregators.

Future work

- Incorporating uncertainties into the model of aggregators.
- Beyond linear bidding functions.