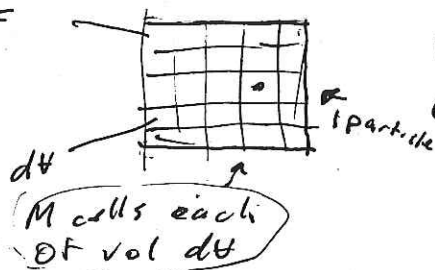


IV) Connecting k to $R = \text{universal gas constant}$

LEC. 4
SPQ 15
Appendix 1

(1)

A) Vol = V



$$V = M \Delta V$$

⊕ 1 particle in M cells of dV

dV can be on order

$$\text{of } d_0^3 \Rightarrow M \approx V/d_0^3$$

$d_0 = \text{atomic diameter}$

⊕ no. of ways of placing or observing

$$\text{this ptcl.} = M = \frac{V}{\Delta V}$$

⊕ in each dV , the ptcl. can be observed to be in any of the energy states avail to it.

⊕ S.O, no. of ways of placing or observing N ptcls, each and all of which have no interaction w/ the others is

thus ;
$$\begin{matrix} * & M \cdot M \cdot & \dots & M \\ * & \text{1 ptcl 2nd ptcl} & & \text{Nth ptcl} \end{matrix} = M^N = \frac{V^N}{\Delta V^N}$$

⊕ This argument neglects the volume of each of the N ptcls.

⊕ Thus, the number of microstates avail to the N -particle, non-interacting system, $\Omega = \Omega(E, V, N) \propto \frac{V^N}{\Delta V^N}$ as claimed by pathria.

B) THUS, @ EQUILIBRIUM, SINCE

LCC &
SPG15
APP1

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$$\eta \equiv \frac{\partial \ln \Omega}{\partial \mu} \bigg|_{E, N}$$

and

$$\frac{\partial S}{\partial \mu} \bigg|_{E, N} = \frac{P}{T}$$

and

$$S = k \ln \Omega$$

then, $\frac{\partial S}{\partial \mu} \bigg|_{E, N} \stackrel{\text{ALSO}}{=} k \eta = \frac{P}{T} \Rightarrow \boxed{\eta = \frac{P}{kT}}$

(As shown before)

C) NEXT, SINCE $\Omega \propto N$

ARGUMENT 1

$$\frac{\partial \ln \Omega}{\partial \mu} \bigg|_{E, N} \propto \frac{N}{\mu}$$

or

$$\eta \propto \frac{N}{\mu}$$

or

$$\frac{P}{kT} \propto \frac{N}{\mu}$$

or

$$\boxed{k \propto \frac{P\mu}{NT}}$$

ARGUMENT 2

i) $\Omega = f(E) N$
(i.e., recognizing that proportionality constant is actually a fcn of E.)

ii) $\ln \Omega = N(\ln \mu + \ln f(E))$

iii) $\therefore \frac{\partial \ln \Omega}{\partial \mu} \bigg|_{E, N} = \frac{N}{\mu}$

iv) or since

$$\frac{\partial \ln \Omega}{\partial \mu} \bigg|_{E, N} = \eta = \frac{P}{kT}$$

THEN

$$\boxed{\frac{P}{kT} = \frac{N}{\mu}} \quad (*)$$

v) comparing (*) w/ IDEAL GAS LAW: $P\mu = n\bar{R}T$
we get a relationship between k & $\bar{R}$:
 $kN = n\bar{R}$

APPL (3)

OR
$$K = \frac{n\bar{R}}{N} = \frac{N}{A_0} \frac{\bar{R}}{N} = \frac{\bar{R}}{A_0}$$

where A_0 = Avogadro's number

Office 4 SDG 15
Rough Notes

SCALING / QUALITATIVE

$\Omega(E, V, N)$ ARGUMENT Regarding

THE APPROX. DEPENDENCE OF

$\Omega = \Omega(E, V, N)$ ON E

(A)

@ Ref: Reif, Fundamentals of
Statistical Thermo, PP. 61-63

1) CONSIDER A (QUANTUM)

SYSTEM OF N ptcls. LET
(QUANTUM)

THE[^] DYNAMICS OF A SINGLE

Ptcl BE CHARACTERIZED BY SAY M
DEGREES OF FREEDOM. FOR EXAMPLE,

~~LET $\phi_i(t)$ BE~~

THE FORCE-FREE MOTION

OF A SINGLE ptcl, say an

electron (or a proton, or an atom, or
a photon, etc), in a "Box"

(= an enclosed space

bounded by an effectively
infinite potential) IS

characterized by 3 d.o.f. -

translation in the 3-coord.

directions.

Stargh

1) $n = \phi_r(E) \sim E^q$

$n = h\omega(n + \frac{1}{2})$

2) for $\neq$ dot

$\Phi_+ = \Phi_1^+ = E^+$

$E \sim E/p$

3) $\Omega \cong \mathcal{H} \mathcal{O} \mathcal{F}$

$\Phi_r(E + dE) + \Phi_e(E)$

$= \frac{\partial \Phi_r}{\partial E} \delta E$

$= \frac{\partial}{\partial E} (\Phi_1^+) \delta E \cong \frac{\partial \Phi_1^+}{\partial E} \left(\frac{1}{E} \right) M(E/p)$

$\cong \Phi_1^+ M$

$\therefore \textcircled{1} \Omega(E, V, N) \sim \Phi_1^+$

$\textcircled{2} \ln \Omega \sim O(E) \sim O(N)$

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$\textcircled{1} a) \neq p^{\text{rel}} \quad \neq$

b) $M = V/\sigma V$

c) $\Omega_{\text{rel}}(N=1, E, \neq) = f(E) M^{1/\sigma V}$

d) $\therefore \Omega(N, E, \neq) \propto F(E) V^N$

e) $\Rightarrow \ln \Omega = N \ln V + \ln F(E)$
 $\ln \Omega / N = \ln V + \ln F(E) / N$
 $\ln \Omega / N = \ln V + \ln F(E) / N$
 $\ln \Omega / N = \ln V + \ln F(E) / N$

$\textcircled{2} N k T = p V$
 $p \bar{v} = p V$

$N/A_0 \Rightarrow \bar{v} = \bar{v}/A_0$

(Handwritten notes and corrections in the right margin)

PCG TMR

$\mathcal{N} \sim E^{\alpha}$

with

Rough / Incomplete

(22)

i) N ptls. $\Rightarrow$ ~~not~~ d.o.f. $\sim \mathcal{N}$ free ptls
 in $\Phi(E) \equiv$ no. of quant states
 for a single d.o.f. (eg 1-comp. or
 translation)

ii) ~~PCG~~ arg's that

$$\Phi(E) \sim E^{\alpha}$$

$$\alpha < \mathcal{O}(1)$$

a) $\uparrow$ Quantum Osc.

$$E_n = \hbar \omega (n + 1/2)$$

$$\Rightarrow \text{here } \Phi(E) = \Phi(E_n) = n$$

$$\therefore \boxed{\Phi(E) \propto E(1)}$$

b) F q.d. ptls in 1-D

$$E_n^{(1D)} = \left(\frac{\hbar^2}{8mL^2} \right)^{1/2} n^2$$

$$\Rightarrow \Phi(E) = n_x$$

$$\therefore \Phi \propto E^{1/2}$$

$$\text{or } \boxed{\alpha = 1/2}$$

iii) For F total d.o.f. and N ptls each w/ charact energy ϵ_i , there are $\Phi(E)$ possible states that can be ~~the~~ assigned by 1st d.o.f. $\Phi_1(E)$ for the 2nd d.o.f. $\Phi_2(E)$
 $\Rightarrow \mathcal{N}(E) \approx (\Phi(E))^F$

Rough

no

or $\boxed{\Omega(E, N) \approx e^{\beta}}$

or next since $E \sim E/f$

then $\boxed{\Omega(E, N) \approx E^f}$

$$\Omega(E, N, V) \approx \Omega(E + dE, N, V) - \Omega(E, N, V)$$

$$= \frac{\partial \Omega}{\partial E} \bigg|_E dE$$

$$\frac{\partial \Omega}{\partial E} \bigg|_E \approx f E^{f-1}$$