

2.5 Defining Functions in the λ -notation

Elwood Warren-Kuelgen

June 18, 2026

Exercises:

1. Describe the following functions in words:
 - (a) $\lambda x \in \mathbb{N}. x > 3 \text{ and } x < 7$
The function which maps every x such that x is a natural number to 1 if x is between 3 and 7, and to 0 otherwise.
 - (b) $\lambda x : x \text{ is a person. } x\text{'s father}$
The function which maps everyone to their father.
 - (c) $\lambda X \in Pow(D). \{y \in D : y \notin X\}$
The function which maps every subset of D to its complement in D .
 - (d) $\lambda X \subseteq D. [\lambda y \in D. y \notin X]$
The function which maps every X such that X is a subset of the domain to a function which maps members of the domain to 1 if they are not in X .
2. In this exercise, simple functions are described in a complicated way. Simplify them as much as possible.
 - (a) $[\lambda x \in D. [\lambda y \in D. [\lambda z \in D. z \text{ introduced } x \text{ to } y]]](\text{Ann})(\text{Sue})$
 $[\lambda x. [\lambda y. [\lambda z. z \text{ introduced } x \text{ to } y]]](\text{Ann})(\text{Sue})$
 $\lambda x. x \text{ introduced Ann to Sue}$
 - (b) $[\lambda x \in D. [\lambda y \in D. [\lambda z \in D. z \text{ introduced } x \text{ to } y](\text{Ann})](\text{Sue})$
 $\lambda x. \text{Ann introduced } x \text{ to Sue}$
 - (c) $[\lambda x \in D. [\lambda y \in D. [\lambda z \in D. z \text{ introduced } x \text{ to } y](\text{Ann})]](\text{Sue})$
 $\lambda x. \text{Ann introduced Sue to } x$
 - (d) $[\lambda x \in D. [\lambda y \in D. [\lambda z \in D. z \text{ introduced } x \text{ to } y]](\text{Ann})](\text{Sue})$
 $\lambda x. x \text{ introduced Sue to Ann}$
 - (e) $[\lambda f \in D_{<e,t>}. [\lambda x \in D_e. f(x) = 1 \text{ and } x \text{ is a gray}]]([\lambda y \in D_e. y \text{ is a cat}])$
 $\lambda x \in D_e. x \text{ is a cat and } x \text{ is gray}$
 - (f) $[\lambda f \in D_{<e,<e,t>>}. [\lambda x \in D_e. f(x)(\text{Ann}) = 1]]([\lambda y \in D_e. [\lambda z \in D_e. z \text{ saw } y]])$
 $\lambda x \in D_e. \text{Ann saw } x$

- (g) $[\lambda x \in \mathbb{N}. [\lambda y \in \mathbb{N}. 3 < y < 7](x)]$
 $\lambda x \in \mathbb{N}. 3 < x < 7$
- (h) $[\lambda z \in \mathbb{N}. [\lambda y \in \mathbb{N}. [\lambda x \in \mathbb{N}. 3 < x < 7](y)](z)]$
 $\lambda x \in \mathbb{N}. 3 < x < 7$

3. Suppose "and" and "or" have Schönfinkled denotations, that is $\llbracket \mathbf{and} \rrbracket$ and $\llbracket \mathbf{or} \rrbracket$ are both members of $D_{\langle t, \langle t, t \rangle \rangle}$. They are functions that map truth-values into functions from truth-values to truth-values. Specify the two functions using the λ -notation.

- (a) $\llbracket \mathbf{and} \rrbracket = \lambda x \in D_t. [\lambda y \in D_t. x = y = 1]$
- (b) $\llbracket \mathbf{or} \rrbracket = \lambda x \in D_t. [\lambda y \in D_t. x = 1 \text{ or } y = 1]$

4. Replace the "?" in each of the following statements:

- (a) $[\lambda f \in D_{\langle e, t \rangle}. [\lambda x \in D_e. f(x) = 1 \text{ and } x \text{ is gray}]] \in D_?$
 $D_? = D_{\langle \langle e, t \rangle, \langle e, t \rangle \rangle}$
- (b) $[\lambda f \in D_{\langle e, \langle e, t \rangle \rangle}. [\lambda x \in D_e. f(x)(\text{Ann}) = 1]] \in D_?$
 $D_? = D_{\langle \langle e, \langle e, t \rangle, \langle e, t \rangle \rangle \rangle}$
- (c) $[\lambda y \in D_e. [\lambda f \in D_{\langle e, t \rangle}. [\lambda x \in D_e. f(x) = 1 \text{ and } x \text{ is in } y]]] \in D_?$
 $D_? = D_{\langle e, \langle \langle e, t \rangle, \langle e, t \rangle \rangle \rangle}$
- (d) $[\lambda f \in D_{\langle e, t \rangle}. \text{there is some } x \in D_e \text{ such that } f(x) = 1] \in D_?$
 $D_? = D_{\langle \langle e, t \rangle, t \rangle}$
- (e) $[\lambda f \in D_{\langle e, t \rangle}. \text{Mary}] \in D_?$
 $D_? = D_{\langle \langle e, t \rangle, e \rangle}$
- (f) $[\lambda f \in D_{\langle e, t \rangle}. [\lambda g \in D_{\langle e, t \rangle}. \text{there is no } x \in D_e \text{ such that } f(x) = 1 \text{ and } g(x) = 1]] \in D_?$
 $D_? = D_{\langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle}$

Chapter 4: More of English

Nonverbal Predicates, Modifiers, Definite Descriptions

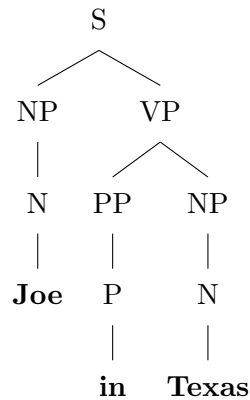
Elwood Warren-Kuelgen

June 18, 2026

4.2 Nonverbal Predicates

Exercise: Calculate the truth-conditions for at least one of the sentences "Joe is in Texas", "Joe is fond of Kaline", and "Kaline is a cat".

Derivation of $\llbracket \text{Joe is in Texas} \rrbracket$:

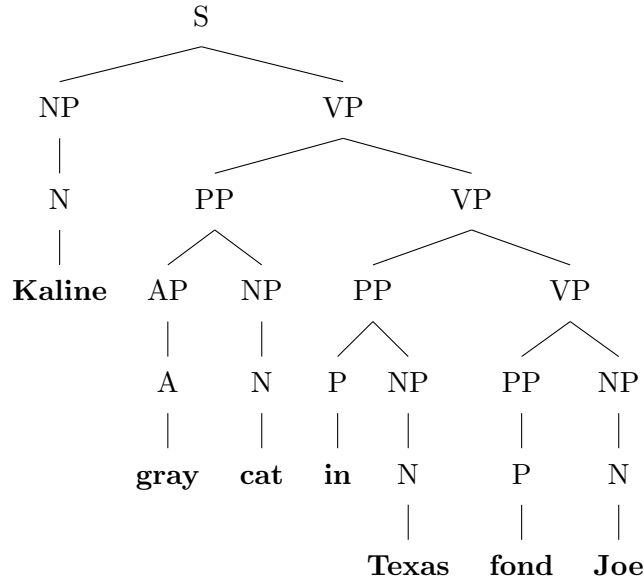


1. By FA: $\llbracket \text{Joe in Texas} \rrbracket = \llbracket \text{in Texas} \rrbracket (\llbracket \text{Joe} \rrbracket)$
2. By NN & the lexical entry for **Joe**: $\llbracket \text{Joe} \rrbracket = \text{Joe}$
3. Hence: $\llbracket \text{in Texas} \rrbracket (\llbracket \text{Joe} \rrbracket) = \llbracket \text{in Texas} \rrbracket (\text{Joe})$
4. By FA: $\llbracket \text{in Texas} \rrbracket = \llbracket \text{in} \rrbracket (\llbracket \text{Texas} \rrbracket)$
5. By FA: $\llbracket \text{in Texas} \rrbracket = \llbracket \text{in} \rrbracket (\llbracket \text{Texas} \rrbracket) \llbracket \text{in} \rrbracket (\llbracket \text{Texas} \rrbracket) = \llbracket \text{in} \rrbracket (\text{Texas})$
6. By NN & the lexical entry for **in**: $\llbracket \text{in} \rrbracket (\text{Texas})$
 $= [\lambda x \in D_e. [\lambda y \in D_{\langle e, t \rangle}. y \text{ is in } x]](\text{Texas}) = \lambda y \in D_{\langle e, t \rangle}. y \text{ is in Texas}$
7. Hence, $\llbracket \text{in Texas} \rrbracket (\text{Joe}) = [\lambda y \in D_{\langle e, t \rangle}. y \text{ is in Texas}] (\text{Joe})$

4.3.1 A New Composition Rule

Calculate the truth-conditions for (8), given one possible syntactic parse.

Derivation of $\llbracket \text{Kaline is a gray cat in Texas fond of Joe} \rrbracket$:

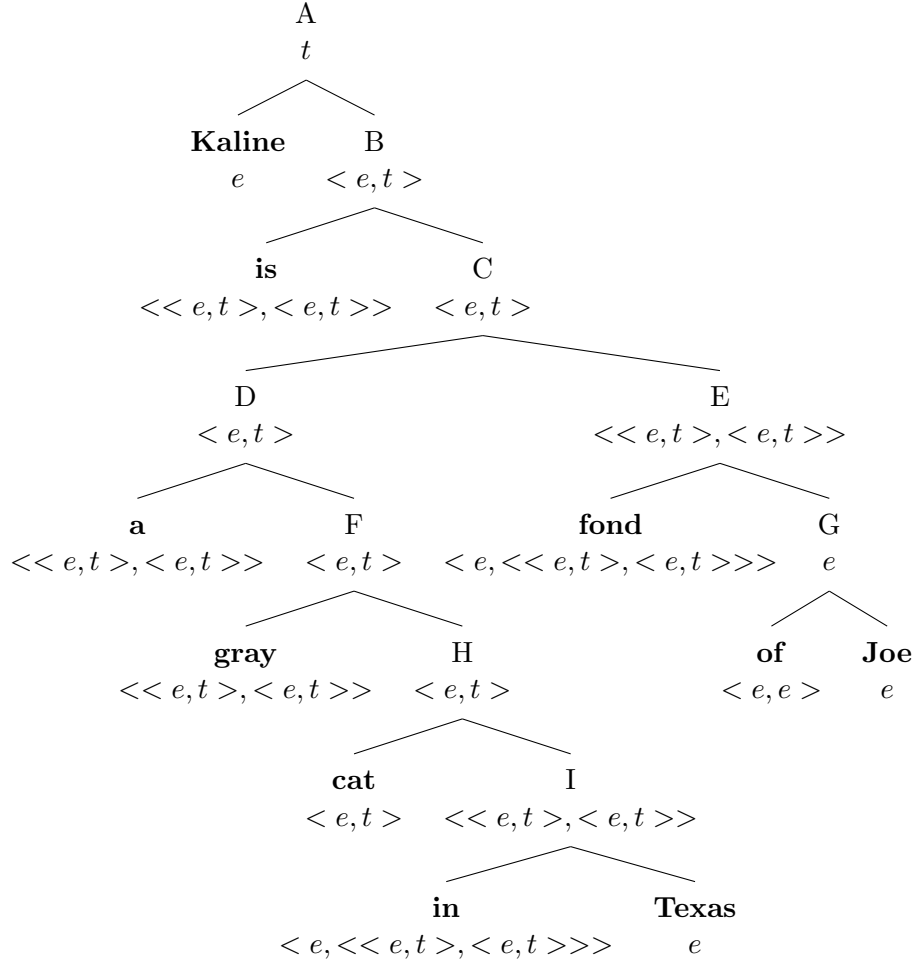


1. New Lexical Entries:
 - a) $\llbracket \text{Gray} \rrbracket = \lambda x \in D_e. x \text{ is gray}$
 - b) $\llbracket \text{Cat} \rrbracket = \lambda x \in D_e. x \text{ is a cat}$
 - c) $\llbracket \text{In} \rrbracket = \lambda x \in D_e. [\lambda y \in D_e. y \text{ is in } x]$
 - d) $\llbracket \text{Fond} \rrbracket = \lambda x \in D_e. [\lambda y \in D_e. y \text{ is fond of } x]$
2. By FA: $\llbracket \text{Kaline is a gray cat in Texas Fond of Joe} \rrbracket = \llbracket \text{gray cat in Texas Fond of Joe} \rrbracket(\llbracket \text{Kaline} \rrbracket)$
3. By PM: $\llbracket \text{gray cat in Texas Fond of Joe} \rrbracket = \llbracket \text{in Texas Fond of Joe} \rrbracket(\llbracket \text{Gray Cat} \rrbracket)$
4. By PM: $\llbracket \text{in Texas fond of Joe} \rrbracket = \llbracket \text{in Texas} \rrbracket(\llbracket \text{fond of Joe} \rrbracket)$
 . . .
5. Keep applying FA, PM, and the lexical Entries to yield:
 $\llbracket \text{gray cat in Texas Fond of Joe} \rrbracket(\text{Kaline}) =$
 $[\lambda x \in D_e. x \text{ is gray and } x \text{ is a cat and } x \text{ is in Texas and } x \text{ is fond of Joe}](\text{Kaline})$

4.3.2 Modification as Functional Application

Exercise 1: Calculate the truth-conditions for (8), given one possible syntactic parse. This time, use FA instead of PM.

Second Derivation of $\llbracket \text{Kaline is a gray cat in Texas fond of Joe} \rrbracket$:



1. Revised Lexical Entries:

- a) $\llbracket \text{In} \rrbracket = \lambda x \in D_e. [\lambda f \in D_{\langle e, t \rangle}. [\lambda y \in D_e. f(y) = 1 \text{ and } y \text{ is in } x]]$
- b) $\llbracket \text{Cat} \rrbracket = \lambda x \in D_e. x \text{ is a cat}$
- c) $\llbracket \text{gray} \rrbracket = \lambda f \in D_{\langle e, t \rangle}. [\lambda x \in D_e. f(x) = 1 \text{ and } x \text{ is gray}]$
- d) $\llbracket \text{fond} \rrbracket = \lambda x \in D_e. [\lambda f \in D_{\langle e, t \rangle}. [\lambda y \in D_e. f(y) = 1 \text{ and } y \text{ is fond of } x]]$

2. **FA:** $\llbracket \text{of Joe} \rrbracket = \llbracket \text{of} \rrbracket(\llbracket \text{Joe} \rrbracket)$
3. **LE:** $= [\lambda x. x](\text{Joe})$
4. **λ -reduction:** $= \text{Joe}$

5. **FA:** $\llbracket \text{fond of Joe} \rrbracket = \llbracket \text{fond} \rrbracket(\llbracket \text{of Joe} \rrbracket)$
6. **LE,** (4): $= [\lambda x \in D_e. [\lambda f \in D_{\langle e, t \rangle}. [\lambda y \in D_e. f(x) = 1 \text{ and } y \text{ is fond of } x]]](\text{Joe})$
7. **λ -reduction:** $= \lambda f \in D_{\langle e, t \rangle}. [\lambda y \in D_e. f(x) = 1 \text{ and } y \text{ is fond of Joe}]$

8. **FA:** $\llbracket \text{in Texas} \rrbracket = \llbracket \text{in} \rrbracket(\llbracket \text{Texas} \rrbracket)$
9. **LE:** $= [\lambda x \in D_e. [\lambda f \in D_{\langle e, t \rangle}. [\lambda y \in D_e. f(y) = 1 \text{ and } y \text{ is in } x]]](\text{Texas})$
10. **λ -reduction:** $= \lambda f \in D_{\langle e, t \rangle}. [\lambda y \in D_e. f(y) = 1 \text{ and } y \text{ is in Texas}]$

11. **FA:** $\llbracket \text{cat in Texas} \rrbracket = \llbracket \text{in Texas} \rrbracket(\llbracket \text{cat} \rrbracket)$
12. **LE:** $= [\lambda f \in D_{\langle e, t \rangle}. [\lambda y \in D_e. f(y) = 1 \text{ and } y \text{ is in Texas}]](\lambda x \in D_e. x \text{ is a cat})$
13. **λ -reduction:** $= \lambda y \in D_e. y \text{ is a cat and } y \text{ is in Texas}$

14. **FA:** $\llbracket \text{gray cat in Texas} \rrbracket = \llbracket \text{gray} \rrbracket(\llbracket \text{cat in Texas} \rrbracket)$
15. **LE:** $= [\lambda f \in D_{\langle e, t \rangle}. [\lambda x \in D_e. f(x) = 1 \text{ and } x \text{ is gray}]](\lambda y \in D_e. y \text{ is a cat and } y \text{ is in Texas})$
16. **λ -reduction:** $= \lambda x \in D_e. x \text{ is a cat and } x \text{ is in Texas and } x \text{ is gray}$

17. **FA:** $\llbracket \text{a gray cat in Texas} \rrbracket = \llbracket \text{a} \rrbracket(\llbracket \text{gray cat in Texas} \rrbracket)$
18. **LE:** $= [\lambda x. x](\lambda x \in D_e. x \text{ is a cat and } x \text{ is in Texas and } x \text{ is gray})$
19. **λ -reduction:** $= \lambda x \in D_e. x \text{ is a cat and } x \text{ is in Texas and } x \text{ is gray}$

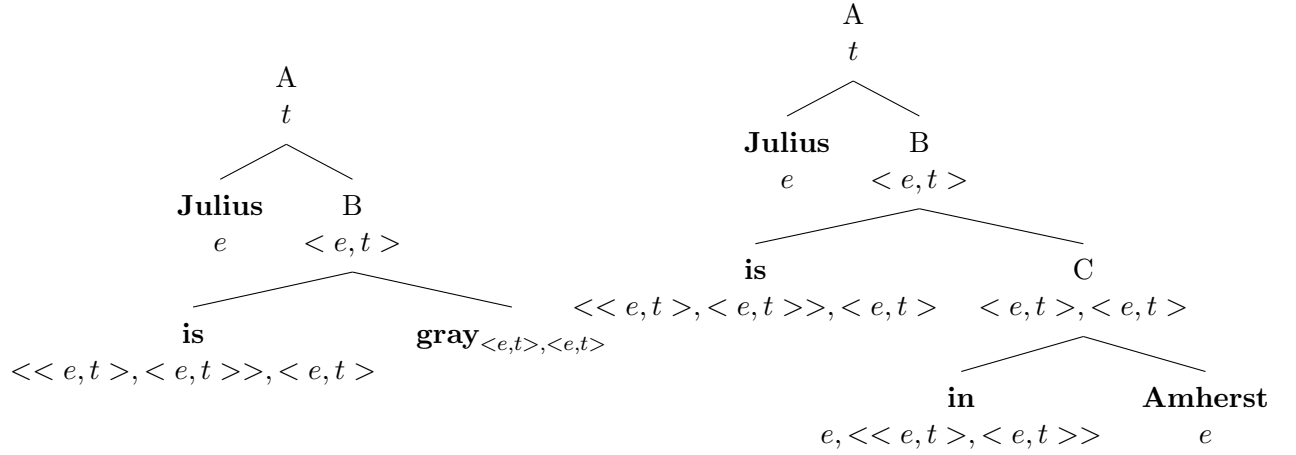
20. **FA:** $\llbracket \text{a gray cat in Texas fond of Joe} \rrbracket = \llbracket \text{fond of Joe} \rrbracket(\llbracket \text{a gray cat in Texas} \rrbracket)$
21. **LE:** $= [\lambda f \in D_{\langle e, t \rangle}. [\lambda y \in D_e. f(x) = 1 \text{ and } y \text{ is fond of Joe}]](\lambda x \in D_e. x \text{ is a cat and } x \text{ is in Texas and } x \text{ is gray})$
22. **λ -reduction:** $= \lambda y \in D_e. y \text{ is a cat and } y \text{ is in Texas and } y \text{ is gray and } y \text{ is fond of Joe}$

23. **FA:** $\llbracket \text{is a gray cat in texas fond of Joe} \rrbracket = \llbracket \text{is} \rrbracket(\llbracket \text{a gray cat in Texas} \rrbracket)$
24. **LE:** $= [\lambda x. x](\lambda y \in D_e. y \text{ is a cat and } y \text{ is in Texas and } y \text{ is gray and } y \text{ is fond of Joe})$
25. **λ -reduction:** $= \lambda y \in D_e. y \text{ is a cat and } y \text{ is in Texas and } y \text{ is gray and } y \text{ is fond of Joe}$

26. **FA:** $\llbracket \text{Kaline is a gray cat in texas fond of Joe} \rrbracket = \llbracket \text{is a gray cat in Texas fond of Joe} \rrbracket(\llbracket \text{Kaline} \rrbracket)$
27. **LE:** $= \lambda y \in D_e. y \text{ is a cat and } y \text{ is in Texas and } y \text{ is gray and } y \text{ is fond of Joe}(\text{Kaline})$
28. **λ -reduction:** $= 1 \text{ iff } \textit{Kaline} \text{ is a cat and } \textit{Kaline} \text{ is in Texas and } \textit{Kaline} \text{ is gray and } \textit{Kaline} \text{ is fond of Joe}$

Exercise 2: If APs and PPs denote functions of type $\langle e, t \rangle, \langle e, t \rangle$, these sentences are prima facie uninterpretable. We could try to solve the problem by assigning a suitable denotation to the copula "be". Define such a denotation. There are two distinct solutions.

Syntax Trees for "Julius is Gray" and "Julius is in Amherst":



1. Solution 1 for $\llbracket \text{is} \rrbracket$ i.e. $\llbracket \text{be} \rrbracket$:
 Assume that $\llbracket \text{is} \rrbracket$ takes members of $D_{\langle e, t \rangle, \langle e, t \rangle}$ as arguments (as shown above).
 $\llbracket \text{is} \rrbracket = \lambda f \in D_{\langle \langle e, t \rangle, \langle e, t \rangle \rangle}. [\lambda g \in D_e. f(\lambda x. 1)]$
2. Solution 2 for $\llbracket \text{is} \rrbracket$:
 Assume that $\llbracket \text{is} \rrbracket \in D_{\langle e, t \rangle}$. Hence, $\llbracket \text{gray} \rrbracket(\llbracket \text{is} \rrbracket) \in D_{\langle e, t \rangle}$.
 $\llbracket \text{is} \rrbracket = \lambda x. 1$

4.4.2 Partial denotations and the distinction between presupposition and assertion

Exercise 1: Consider an example with one definite description embedded in another:

(i) the killer of the black cat escaped.

(a) Draw an interpretable syntactic structure for (i)

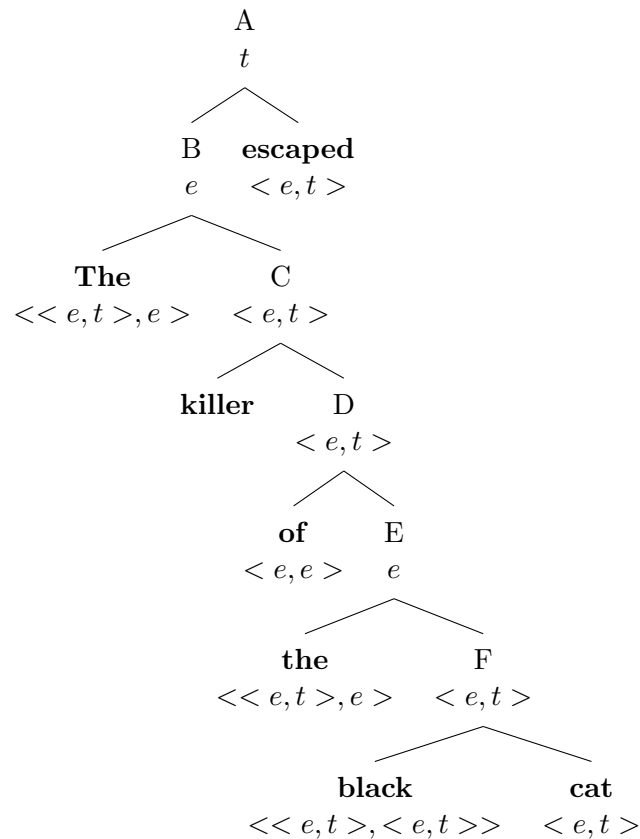
(b) Describe three possible states of affairs:

One where (i) is false.

One where (i) lacks a truth-value because "the black cat" has no extension.

One where (i) is also without a truth-value, but this time because "the killer of the black cat" lacks a denotation. Assume "the black cat" does have a denotation.

Syntax tree for $\llbracket \text{the killer of the black cat escaped} \rrbracket$



1. **State of affairs where (i) is false:**

Its not the case that the individual picked out by $\llbracket \text{the killer of the black cat} \rrbracket$ escaped. Hence, $\llbracket \text{escaped} \rrbracket(\llbracket \text{the killer of the black cat} \rrbracket) = 0$

2. **State of affairs where (i) lacks a truth-value because "the black cat" has**

no extension:

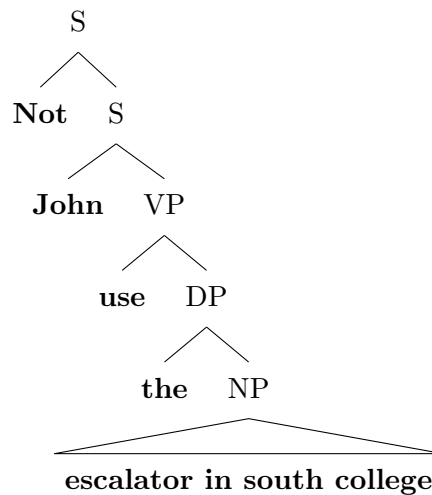
There is no unique individual that is a black cat. Hence, either $\exists x, y \in D_e$ such that $\llbracket \text{black cat} \rrbracket(x) = \llbracket \text{black cat} \rrbracket(y) = 1$ and $x \neq y$, or $\neg \exists x \in D_e$ such that $\llbracket \text{black cat} \rrbracket(x) = 1$. Consequently, $\llbracket \text{black cat} \rrbracket$ is not in the domain of $\llbracket \text{the} \rrbracket$.

3. **State of affairs where (i) lacks a truth-value because "the killer of the black cat" lacks a denotation:**

There is no unique individual that killed the unique black cat. Hence, either $\exists x, y \in D_e$ such that $\llbracket \text{killer of the black cat} \rrbracket(x) = \llbracket \text{killer of the black cat} \rrbracket(y) = 1$ and $x \neq y$, or $\neg \exists x \in D_e$ such that $\llbracket \text{killer of the black cat} \rrbracket(x) = 1$. Consequently, $\llbracket \text{killer of the black cat} \rrbracket$ is not in the domain of $\llbracket \text{the} \rrbracket$.

Exercise 2: Assume that sentence (i) has the structure (i') at the level at which it is interpreted. What does our current theory predict about this sentence? Are the predictions empirically correct? Construct suitable examples and scenarios that help sharpen intuitions and promote discussion.

- (i) John doesn't use the escalator in South College.
(i')**



- 1. What does our current theory predict about this sentence?**

On our current theory, (i') presupposes that there is a unique escalator in south college. Otherwise, it was argued, the speaker would not utter (i), but instead something like "There is a unique escalator in South College, and John does not use it."

Given that there is an escalator in south college, $\llbracket \text{the} \rrbracket (\llbracket \text{escalator in south college} \rrbracket)$ = the unique escalator in south college. (i') is only true if John does not use the escalator in south college.

- 2.**

Exercise 2: Look at the scenario below and consider the following three definite descriptions:

(← LEFT)       (RIGHT →)
 b_1 b_2 b_3 b_4 b_5 b_6

- (i) The leftmost apple in the row.
- (ii) The leftmost dark apple in the row.
- (iii) The apple that is both leftmost in the row and dark.

1. Which individual, if any, does each of these definite descriptions refer to in this situation?

- (i) intuitively refers to b_1
- (ii) ... to b_2
- (iii) ... does not refer.

2. What predictions concerning the denotations of the definite descriptions (i), (ii), and (iii) would follow from an analysis that treats adjectives like "leftmost" as 1-place predicates?

If "leftmost" is treated as a 1-place predicate (that is, if $\llbracket \text{leftmost} \rrbracket$ is treated as an intersective adjective), then the denotation of (i) is the unique object that is both leftmost and an apple; the denotation of (ii) is the unique object that is both leftmost, dark, and an apple; and the denotation of (iii) is equivalent to the denotation of (ii).

On this analysis, only (i) refers; (ii) and (iii) do not have a truth value because there is no unique object satisfying those descriptions.

3. Specify a more adequate denotation for attributive uses of "leftmost"

On a non-intersective reading of $\llbracket \text{leftmost} \rrbracket$:

$\llbracket \text{leftmost} \rrbracket = \lambda f \in D_{\langle e, t \rangle}. [\lambda x \in D_e. f(x) = 1 \text{ and } x \text{ is the leftmost member of } \{y : f(y) = 1\}]$

4. In a compositional fashion, compute the denotation of the definite description "the leftmost dark apple in the row", given the above scenario. For the purpose of this computation, take "apple in the row" as an unanalyzed predicate.

1. FA: $\llbracket \text{dark apple in the row} \rrbracket = \llbracket \text{dark} \rrbracket(\llbracket \text{apple in the row} \rrbracket)$
2. LE: $[\lambda x \in D_{\langle e, t \rangle}. [\lambda y \in D_e. f(y) = 1 \text{ and } y \text{ is dark}]](\lambda x. x \text{ is an apple in the row.})$
3. λ -reduction: $\lambda y. y \text{ is an apple in the row and } y \text{ is dark}$
4. FA: $\llbracket \text{leftmost dark apple in the row} \rrbracket = \llbracket \text{leftmost} \rrbracket(\llbracket \text{dark apple in the row} \rrbracket)$
5. LE: $[\lambda f \in D_{\langle e, t \rangle}. [\lambda x \in D_e. f(x) = 1 \text{ and } x \text{ is the leftmost member of } \{y : f(y) = 1\}]](\lambda x. x \text{ is an apple in the row and } x \text{ is dark})$

6. λ -reduction: $\lambda x. x$ is an apple in the row and is dark and
 x is the leftmost member of $\{y: y \text{ is an apple in the row and } y \text{ is dark}\}$
7. FA: $\llbracket \text{The leftmost dark apple in the row} \rrbracket =$
 $\llbracket \text{The} \rrbracket (\llbracket \text{leftmost dark apple in the row} \rrbracket)$
8. LE: $[\lambda f \in D_{\langle e, t \rangle} \text{ and there is exactly one } x \text{ such that } f(x) = 1.$
the unique y such that $f(y) = 1](\lambda x. x \text{ is an apple in the row and is dark and}$
 $x \text{ is the leftmost member of } \{y: y \text{ is an apple in the row and } y \text{ is dark}\})$
9. λ -reduction: b_2