

## Exam 5 Topics

Exam 5 will cover the topics listed below, from Sections 8.3, 8.4, 8.5 and 8.7 (Taylor polynomials only).

### Important Concepts/Topics

- The Comparison and Limit Comparison Tests
  - See [Comparison Tests - Writing Complete Solutions](#) for the expectations for a full solution on an exam
  - See the [Comparison Big Picture Activity](#) (and [solutions](#)) for practice choosing  $b_n$
- The Alternating Series Test
- The Ratio Test
- Classifying series as absolutely convergent, conditionally convergent or divergent (see [notes](#) for examples)
- Alternating Series Remainder Estimate (formula included on the exam, see box below)
- Integral Test Remainder Estimate (formula included on the exam, see box below)
- Taylor polynomials (The formula is given below for reference. See [notes](#) for examples.)
- Power series - finding the radius and interval of convergence ([notes](#))

In addition, you should know all previous convergence tests (i.e. Divergence Test, Geometric Series Test, p-series Test, Integral Test). **All of our series tests are listed on the next page, for reference.**

The  $n$ th degree Taylor polynomial centered at  $a$  is

$$T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f^{(4)}(a)}{4!}(x-a)^4 + \cdots = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!}(x-a)^k$$

The **Taylor coefficients** are the coefficients that appear in the Taylor polynomial, i.e.

$$C_k = \frac{f^{(k)}(a)}{k!}.$$

The remainder estimates in this box **will be included on the exam**. You do not have to memorize them.

#### Remainder Estimate for the Alternating Series Test:

Suppose  $S = \sum a_k$  is a convergent alternating series and  $a_k = (-1)^k b_k$  or  $a_k = (-1)^{k+1} b_k$  where  $b_k \geq 0$  for each  $n$ . Then the remainder  $R_n = |S - S_n|$  satisfies

$$|R_n| = |S - S_n| \leq b_{n+1}.$$

#### Remainder Estimate for the Integral Test:

Suppose  $f(k) = a_k$  where  $f(x)$  is a continuous, positive, decreasing function and  $\sum a_k$  is convergent. Then the remainder  $R_n = |S - S_n|$  satisfies

$$\int_{n+1}^{\infty} f(x) dx \leq R_n \leq \int_n^{\infty} f(x) dx.$$

**The Divergence Test:** If  $\lim_{n \rightarrow \infty} a_n \neq 0$ , then  $\sum a_n$  diverges.

Note: You can only conclude *divergence* using the Divergence Test. If  $\lim_{n \rightarrow \infty} a_n = 0$ , the test is *inconclusive*.

**The Geometric Series Test:** This test only applies to geometric series, i.e. series of the form  $\sum_{n=1}^{\infty} ar^{n-1}$ . This series converges if  $|r| < 1$  and diverges if  $|r| \geq 1$ .

Note: If the series converges, then we can compute the value using the formula  $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ .

**The  $p$ -Series Test:** This test only applies to  $p$ -series, i.e. series of the form  $\sum_{n=1}^{\infty} \frac{1}{n^p}$ . A  $p$ -series converges if  $p > 1$  and diverges if  $p \leq 1$ .

**The Integral Test:** Given  $\sum a_n$ , if  $f(n)$  is a function such that  $f(n) = a_n$  for each  $n$  and  $f$  is continuous, positive and decreasing, then

$$\sum_{n=1}^{\infty} a_n \text{ converges if and only if } \int_1^{\infty} f(x) dx \text{ converges.}$$

**The (Direct) Comparison Test:** Suppose  $\sum a_n$ ,  $\sum b_n$  are series with positive terms.

- If  $\sum b_n$  is convergent and  $a_n \leq b_n$  for each  $n$ , then  $\sum a_n$  is also convergent.
- If  $\sum b_n$  is divergent and  $b_n \leq a_n$  for each  $n$ , then  $\sum a_n$  is also divergent.

**The Limit Comparison Test:** Suppose  $\sum a_n$  and  $\sum b_n$  are series with positive terms. If

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c \quad \text{where } c \text{ is positive and finite}$$

then either both series converge or both diverge.

**The Alternating Series Test:** This test applies only to alternating series, i.e. series that can be written  $\sum (-1)^n b_n$  or  $\sum (-1)^{n+1} b_n$  where  $b_n > 0$ . If

- $\lim_{n \rightarrow \infty} b_n = 0$ , and
- $\{b_n\}$  is decreasing,

then the series is convergent.

Note: If one of the two conditions above is not true, Alternating Series Test is *inconclusive*. However, if the Alternating Series Test fails, the Divergence Test is a good idea to try next.

**The Ratio Test:** Suppose we have a series  $\sum a_n$  and

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L.$$

- If  $L < 1$ , the series is absolutely convergent (hence convergent).
- If  $L > 1$ , the series is divergent.
- If  $L = 1$ , the test is inconclusive.