

Research Agenda

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1 Overview

My primary research interest is on the study of nonlinear conservation laws in partial differential equations (PDE). PDEs are an important branch of mathematics and are widely utilized to model physical phenomenon found in nature. Specifically, my research is based on analyzing PDE models with application in chemotaxis (biology) and traffic flow.

2 Past Research and Ongoing Work

My PhD research was conducted under the kind guidance of my advisor, Professor Tong Li. I am currently writing my PhD dissertation on my study of nonlinear conservation laws.

2.1 Chemotaxis

For the first research work, we solved the Riemann Problem for a model from chemotaxis. We were able to overcome a major obstacle since our system had a non-strictly hyperbolic boundary. In certain regions, we were able to find shock waves and rarefaction waves in the process. Our solutions describe the behavior of organisms due to change in chemical concentration. This work was published in [3].

2.2 Traffic Flow

For the second problem, we investigated a traffic flow model. We were able to show the existence of bounded variation solutions for a concave fundamental diagram. These solutions are important in the context for the traffic flow model to be stable and physically relevant. This was a neat result and a building block for my main PhD topic. This research was presented at the *International Conference on Hyperbolic Problems: Theory, Numerics, and Applications (HYP2022)* in Malaga, Spain.

We extended these results for a traffic flow model with nonconcave fundamental diagrams. These types of models have been observed in experimental data from real-life traffic flow. Our research is significant because this is an open problem in the field. We were able to show this by small perturbation analysis. This research has been published in [2].

3 Problems to Study

In the future, I plan on studying the following problems: (1) Transitional Waves, (2) Mixed-Type problems, and (3) showing existence of global BV solutions with large perturbation.

While the perturbation question stems from traffic flow research, transitional waves and mixed-type problems originate from our work with the chemotaxis model.

3.1 Transitional Waves

Transitional shock waves are shock discontinuities that do not satisfy the Lax characteristic criterion, but satisfy the viscosity admissibility criterion. The chemotaxis system we studied earlier fits the general form from transitional wave theory, and we wish to analyze the behavior of this model from the lens of transitional waves. I approached this problem by experimenting with different viscosity matrices and programmed it in Mathematica. The challenge is to find a specific viscosity matrix that satisfies all the necessary conditions.

3.2 Mixed Type

We also wish to understand the physical meaning of our chemotaxis system as the phase changes from non-strictly hyperbolic to elliptic region. We will utilize the method showed in [1] in which the fundamental wave manifold and the viscosity admissibility criterion were used to analyze shock waves. This may result in homoclinic and heteroclinic orbits. I will approach this problem from the perspective of dynamical systems.

3.3 Traffic Flow

Finally, we would like to further our understanding of traffic flow systems. In the current work described in section 2.2, we show the existence of global BV solutions for nonconcave fundamental diagrams. We estimate the nonlinearity in our traffic flow system by performing small perturbation to a known solution for corresponding concave fundamental diagram. It is natural to study what happens if the nonconcave fundamental diagram is not close to the fundamental diagram. My initial approach will be to add a small term to the equilibrium velocity equation and keep on largening the perturbation until we run into a problem.

Altogether, we wish to expand our earlier research based on PDE models arising from traffic flow and chemotaxis and solve open problems in these areas.

References

- [1] Sunčica Čanić and Bradley J. Plohr. “Shock wave admissibility for quadratic conservation laws”. In: *J. Differential Equations* 118.2 (1995), pp. 293–335. ISSN: 0022-0396. DOI: 10.1006/jdeq.1995.1075. URL: <https://doi-org.proxy.lib.uiowa.edu/10.1006/jdeq.1995.1075>.
- [2] Tong Li and Nitesh Mathur. “Global well-posedness and asymptotic behavior of BV solutions to a system of balance laws”. In: 18.2 (2023), pp. 581–600.
- [3] Tong Li and Nitesh Mathur. “Riemann problem for a non-strictly hyperbolic system in chemotaxis”. In: 27.4 (2022), pp. 2173–2187.