

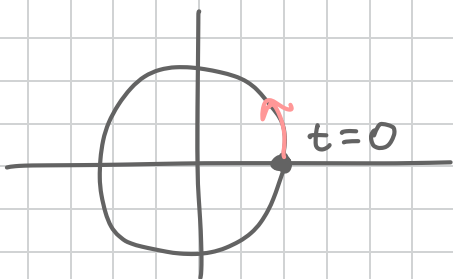
1.7 - Parametric Eqs. ctd.

example:
$$\begin{cases} x = \cos t \\ y = \sin t \end{cases}$$
$$0 \leq t \leq 2\pi$$

★ standard parametrization
of unit circle

$$x^2 + y^2 = (\cos t)^2 + (\sin t)^2 = 1$$

$x^2 + y^2 = 1$ unit circle



starts at $(1,0)$ and goes
once around the circle,
counterclockwise

Variations of Previous Ex:

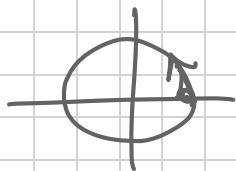
①
$$\begin{cases} x = \cos(2t) \\ y = \sin(2t) \end{cases}$$
$$0 \leq t \leq 2\pi$$

②
$$\begin{cases} x = 2\cos t \\ y = 2\sin t \end{cases}$$
$$0 \leq t \leq 2\pi$$

③
$$\begin{cases} x = \sin t \\ y = \cos t \end{cases}$$
$$0 \leq t \leq \pi$$

④
$$\begin{cases} x = \cos t + 1 \\ y = \sin t - 2 \end{cases}$$
$$-\pi \leq t \leq \pi$$

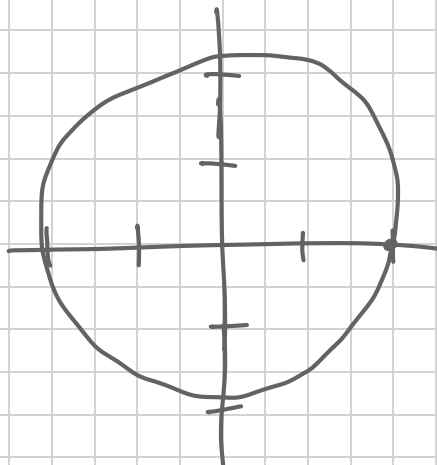
shape, starting & end point, direction?



$$\begin{cases} x = \cos t \\ y = \sin t \\ 0 \leq t \leq 2\pi \end{cases}$$

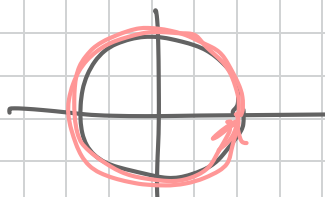
$$\textcircled{2} \begin{cases} x = 2\cos t \\ y = 2\sin t \\ 0 \leq t \leq 2\pi \end{cases}$$

Circle of
radius 2



← makes "time run faster"

$$\textcircled{1} \begin{cases} x = \cos(2t) \\ y = \sin(2t) \\ 0 \leq t \leq 2\pi \end{cases}$$



goes around
twice

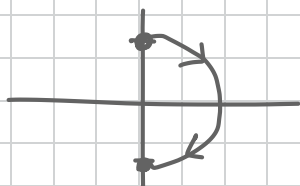
$$\begin{cases} x = \cos t \\ y = \sin t \\ 0 \leq t \leq 4\pi \end{cases}$$

← also goes around
twice

$$\textcircled{3} \begin{cases} x = \sin t \\ y = \cos t \end{cases}$$

$$0 \leq t \leq \pi$$

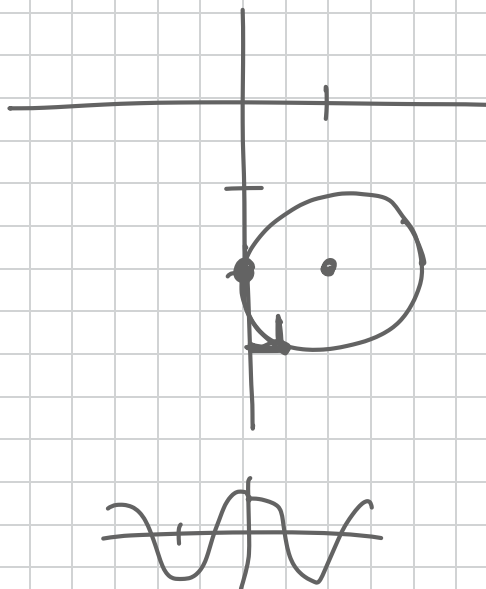
t	x	y
0	0	1
$\pi/2$	1	0
π	0	-1



goes
clockwise

$$\textcircled{4} \begin{cases} x = \cos t + 1 \\ y = \sin t - 2 \end{cases}$$

$$-\pi \leq t \leq \pi$$



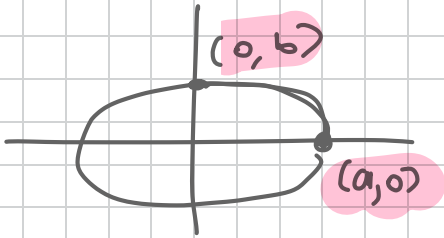
For a circle centered at (a, b) with radius r :

$$\begin{cases} x = r \cos t + a \\ y = r \sin t + b \end{cases}$$

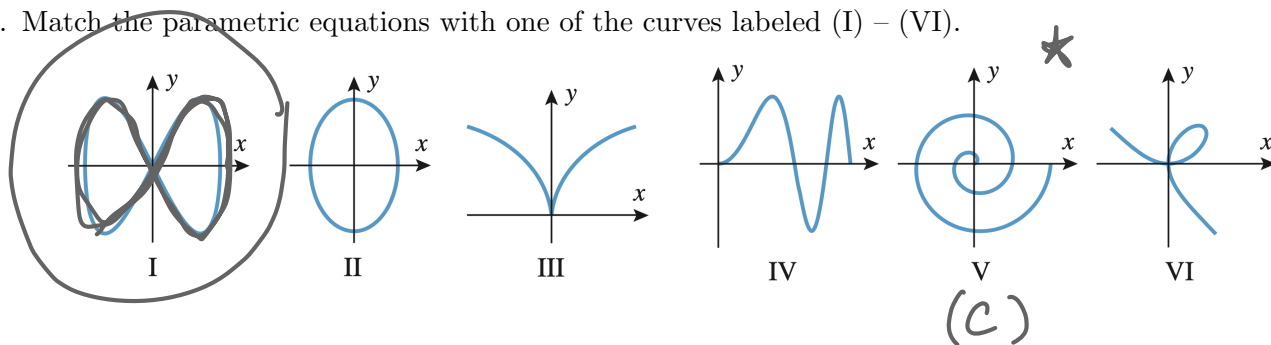
$$0 \leq t \leq 2\pi$$

An ellipse:

$$\begin{cases} x = a \cos t \\ y = b \sin t \end{cases} \quad 0 \leq t \leq 2\pi$$



1. Match the parametric equations with one of the curves labeled (I) – (VI).



(a) $\begin{cases} x = \sqrt{t} \\ y = \sin(3t) \end{cases}$

IV

no interval $\Rightarrow 0 \leq t < \infty$

(d) $\begin{cases} x = \frac{3t}{1+t^3} \\ y = \frac{3t^2}{1+t^3} \end{cases}$

(b) $\begin{cases} x = 2 \cos t \\ y = 3 \sin t \end{cases}$

II

(e) $\begin{cases} x = \frac{t^3}{1+t^2} \\ y = \frac{2t^2}{1+t^2} \end{cases}$

III

(c) $\begin{cases} x = t \cos t \\ y = t \sin t \end{cases}$

V

(f) $\begin{cases} x = \frac{1}{2} \cos t \\ y = \sin(2t) \end{cases}$

★

Strategies: plug in values of t
compare to a previous example

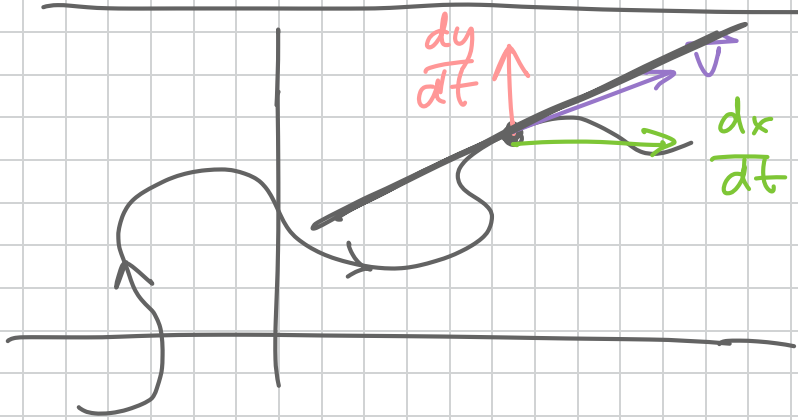
can x or y be negative?

$$\begin{cases} x = \frac{1}{2} \cos t \\ y = \sin(2t) \end{cases}$$

Desmos

$$\begin{cases} x = \cos(at) \\ y = \sin(bt) \end{cases}$$

3.4 - Calculus with Parametric Curves



$\frac{dy}{dt}$ is instantaneous vertical velocity

$\frac{dx}{dt}$ is inst. horizontal velocity

$$\vec{v} = \text{velocity vector} = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$$

We're after the slope and equation of the tangent line:

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt} \right)}{\left(\frac{dx}{dt} \right)}$$

← can pretend these cancel