

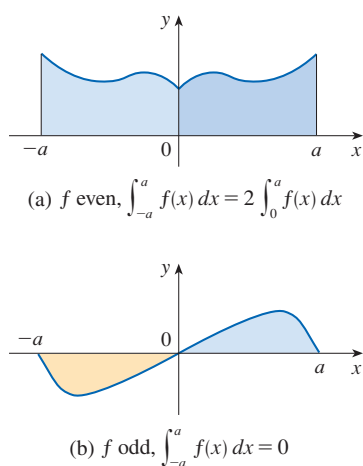


FIGURE 3

(b) If f is odd, then $f(-u) = -f(u)$ and so Equation 9 gives

$$\int_{-a}^a f(x) dx = -\int_0^a f(u) du + \int_0^a f(x) dx = 0$$

Theorem 7 is illustrated by Figure 3. For the case where f is positive and even, part (a) says that the area under $y = f(x)$ from $-a$ to a is twice the area from 0 to a because of symmetry. Recall that an integral $\int_a^b f(x) dx$ can be expressed as the area above the x -axis and below $y = f(x)$ minus the area below the axis and above the curve. So part (b) says the integral is 0 because the areas cancel.

EXAMPLE 10 Because $f(x) = x^6 + 1$ satisfies $f(-x) = f(x)$, it is even and so

$$\begin{aligned} \int_{-2}^2 (x^6 + 1) dx &= 2 \int_0^2 (x^6 + 1) dx \\ &= 2 \left[\frac{1}{7} x^7 + x \right]_0^2 = 2 \left(\frac{128}{7} + 2 \right) = \frac{284}{7} \end{aligned}$$

EXAMPLE 11 Because $f(x) = (\tan x)/(1 + x^2 + x^4)$ satisfies $f(-x) = -f(x)$, it is odd and so

$$\int_{-1}^1 \frac{\tan x}{1 + x^2 + x^4} dx = 0$$

5.5 Exercises

1–8 Evaluate the integral by making the given substitution.

1. $\int \cos 2x dx$, $u = 2x$

2. $\int x e^{-x^2} dx$, $u = -x^2$

3. $\int x^2 \sqrt{x^3 + 1} dx$, $u = x^3 + 1$

4. $\int \sin^2 \theta \cos \theta d\theta$, $u = \sin \theta$

5. $\int \frac{x^3}{x^4 - 5} dx$, $u = x^4 - 5$

6. $\int \frac{1}{x^2} \sqrt{1 + \frac{1}{x}} dx$, $u = 1 + \frac{1}{x}$

7. $\int \frac{\cos \sqrt{t}}{\sqrt{t}} dt$, $u = \sqrt{t}$

8. $\int z \sqrt{z - 1} dz$, $u = z - 1$

9–54 Evaluate the indefinite integral.

9. $\int x \sqrt{1 - x^2} dx$

10. $\int (5 - 3x)^{10} dx$

11. $\int t^3 e^{-t^4} dt$

12. $\int \sin t \sqrt{1 + \cos t} dt$

13. $\int \sin(\pi t/3) dt$

14. $\int \sec^2 2\theta d\theta$

15. $\int \frac{dx}{4x + 7}$

16. $\int y^2 (4 - y^3)^{2/3} dy$

17. $\int \frac{\cos \theta}{1 + \sin \theta} d\theta$

18. $\int \frac{z^2}{z^3 + 1} dz$

19. $\int \cos^3 \theta \sin \theta d\theta$

20. $\int e^{-5r} dr$

21. $\int \frac{e^u}{(1 - e^u)^2} du$

22. $\int \frac{\sin(1/x)}{x^2} dx$

23. $\int \frac{a + bx^2}{\sqrt{3ax + bx^3}} dx$

24. $\int \frac{t + 1}{3t^2 + 6t - 5} dt$

25. $\int \frac{(\ln x)^2}{x} dx$

26. $\int \sin x \sin(\cos x) dx$

27. $\int \sec^2 \theta \tan^3 \theta \, d\theta$ 28. $\int x \sqrt{x+2} \, dx$
29. $\int \left(x - \frac{1}{x^2}\right) \left(x^2 + \frac{2}{x}\right)^5 dx$
30. $\int \frac{dx}{ax+b} \quad (a \neq 0)$
31. $\int e^r(2 + 3e^r)^{3/2} dr$ 32. $\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}} dx$
33. $\int \frac{\sec^2 \theta}{\tan \theta} d\theta$ 34. $\int \frac{\sec^2 x}{\tan^2 x} dx$
35. $\int \frac{(\arctan x)^2}{x^2 + 1} dx$ 36. $\int \frac{1}{(x^2 + 1)\arctan x} dx$
37. $\int 5^t \sin(5^t) dt$ 38. $\int \frac{\sin \theta \cos \theta}{1 + \sin^2 \theta} d\theta$
39. $\int \cos(1 + 5t) dt$ 40. $\int \frac{\cos(\pi/x)}{x^2} dx$
- ~~41. $\int \sqrt{\cot x} \csc^2 x \, dx$~~ 42. $\int \frac{2^t}{2^t + 3} dt$
- ~~43. $\int \sinh^2 x \cosh x \, dx$~~ 44. $\int \frac{dt}{\cos^2 t \sqrt{1 + \tan t}}$
45. $\int \frac{\sin 2x}{1 + \cos^2 x} dx$ 46. $\int \frac{\sin x}{1 + \cos^2 x} dx$
47. $\int \cot x \, dx$ 48. $\int \frac{\cos(\ln t)}{t} dt$
49. $\int \frac{dx}{\sqrt{1-x^2} \sin^{-1} x}$ 50. $\int \frac{x}{1+x^4} dx$
51. $\int \frac{1+x}{1+x^2} dx$ 52. $\int x^2 \sqrt{2+x} \, dx$
53. $\int x(2x+5)^8 dx$ 54. $\int x^3 \sqrt{x^2+1} \, dx$

 **55–58** Evaluate the indefinite integral. Illustrate and check that your answer is reasonable by graphing both the function and its antiderivative (take $C = 0$).

55. $\int x(x^2 - 1)^3 dx$ 56. $\int \tan^2 \theta \sec^2 \theta \, d\theta$
57. $\int e^{\cos x} \sin x \, dx$ 58. $\int \sin x \cos^4 x \, dx$

59–80 Evaluate the definite integral.

59. $\int_0^1 \cos(\pi t/2) \, dt$ 60. $\int_0^1 (3t - 1)^{50} dt$
61. $\int_0^1 \sqrt[3]{1+7x} \, dx$ 62. $\int_{\pi/3}^{2\pi/3} \csc^2(\frac{1}{2}t) \, dt$
63. $\int_0^{\pi/6} \frac{\sin t}{\cos^2 t} dt$ 64. $\int_1^4 \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}} dx$
65. $\int_1^2 \frac{e^{1/x}}{x^2} dx$ 66. $\int_0^1 \frac{e^x}{1+e^{2x}} dx$
67. $\int_{-\pi/4}^{\pi/4} (x^3 + x^4 \tan x) \, dx$ 68. $\int_0^{\pi/2} \cos x \sin(\sin x) \, dx$
69. $\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}$ 70. $\int_0^a x \sqrt{a^2 - x^2} \, dx$
71. $\int_0^a x \sqrt{x^2 + a^2} \, dx \quad (a > 0)$
72. $\int_{-\pi/3}^{\pi/3} x^4 \sin x \, dx$
73. $\int_1^2 x \sqrt{x-1} \, dx$ 74. $\int_0^4 \frac{x}{\sqrt{1+2x}} dx$
75. $\int_e^{e^4} \frac{dx}{x \sqrt{\ln x}}$ 76. $\int_0^2 (x-1)e^{(x-1)^2} dx$
77. $\int_0^1 \frac{e^z + 1}{e^z + z} dz$ 78. $\int_1^4 \frac{1}{(x+1)\sqrt{x}} dx$
79. $\int_0^1 \frac{dx}{(1+\sqrt{x})^4}$ 80. $\int_1^{16} \frac{x^{1/2}}{1+x^{3/4}} dx$

 **81–82** Use a graph to give a rough estimate of the area of the region that lies under the given curve. Then find the exact area.

81. $y = \sqrt{2x+1}, \quad 0 \leq x \leq 1$

82. $y = 2 \sin x - \sin 2x, \quad 0 \leq x \leq \pi$

83. Evaluate $\int_{-2}^2 (x+3)\sqrt{4-x^2} \, dx$ by writing it as a sum of two integrals and interpreting one of those integrals in terms of an area.

84. Evaluate $\int_0^1 x \sqrt{1-x^4} \, dx$ by making a substitution and interpreting the resulting integral in terms of an area.